Equity-Related Volatility Skew

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Equity-Related Volatility Skew

Option Models and the Real World

Modelling the Skew

Skew Risk and Changing Market Conditions

Skew Sensitive Trading Strategies

Skew: What is it?

"Skew" means different things to different people:

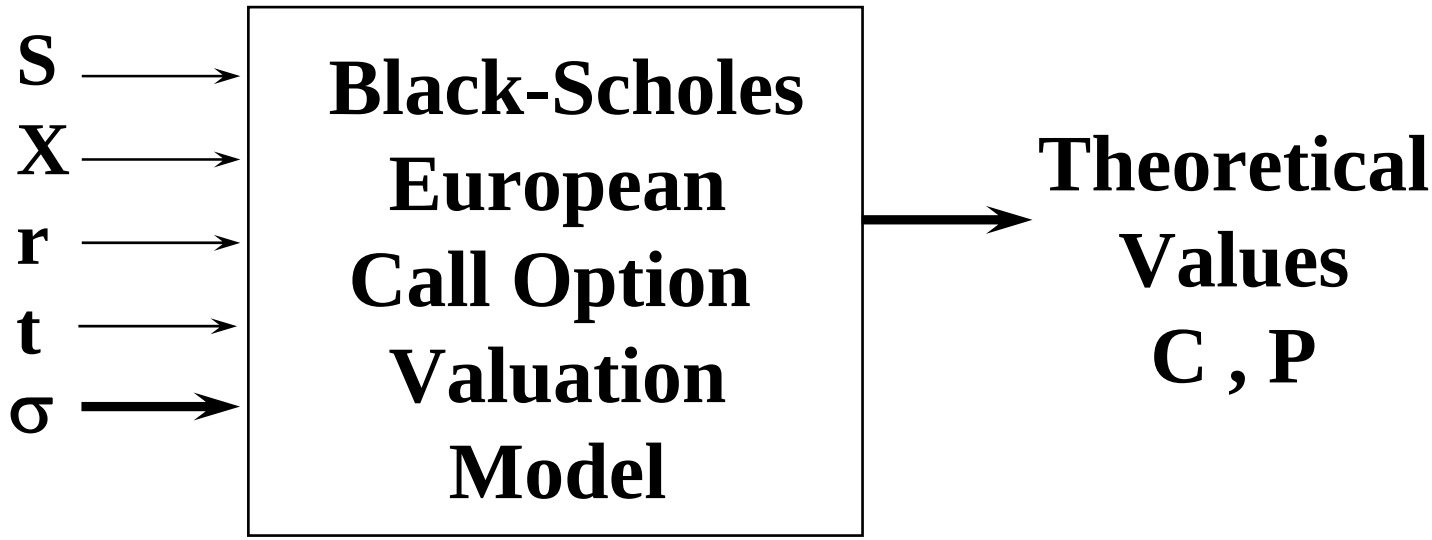
To an Option Trader:

the directional bias (with respect to the underlying asset's price) reflected in option market prices

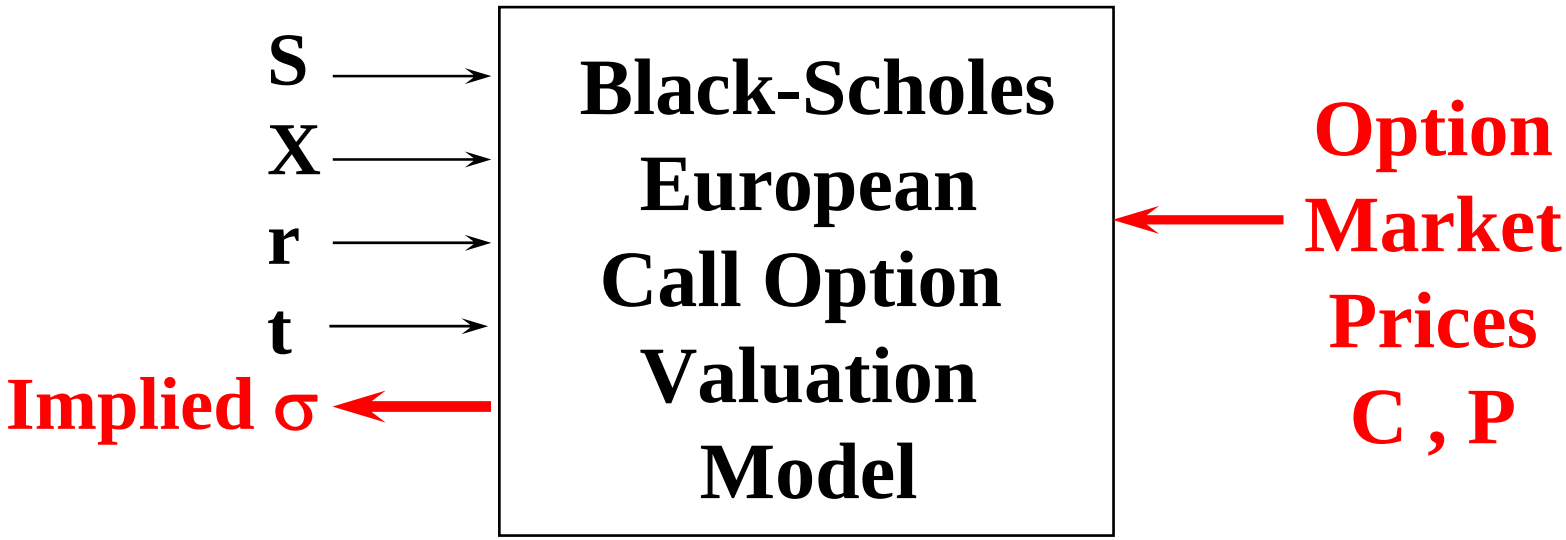
How measured (and relative to what baseline)?

Usually discussed in terms of "implied volatility".

Theoretical Option Valuation

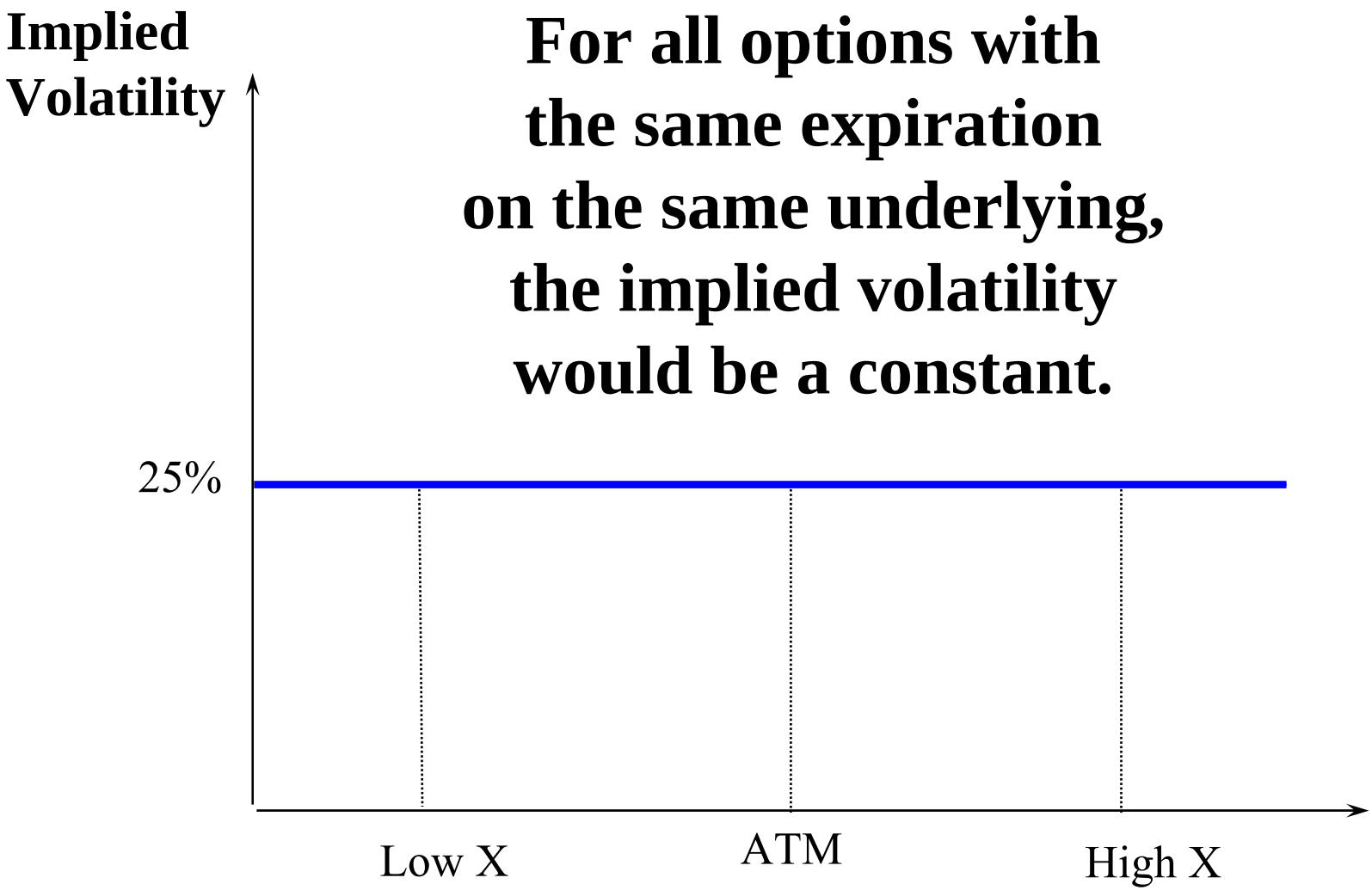


Implied (Market) Volatility



If Black-Scholes were “true”

(that is, if the real world aligned with the assumptions)



Implied Volatility

The CBOE has indicated that

"The average level of the implied volatility curve reflects the average volatility expected by the market."

In this sense, implied volatility (to a certain expiration) can be thought of as the market's expectation of future volatility (between now and that expiration date).

But there are lots of options out there . . .

(obviously with options having different expirations, there can be different implied volatilities, but . . .)

The Volatility "Smile"

Consider an underlying asset (like the S&P 500 or IBM stock). There are many options listed/traded.

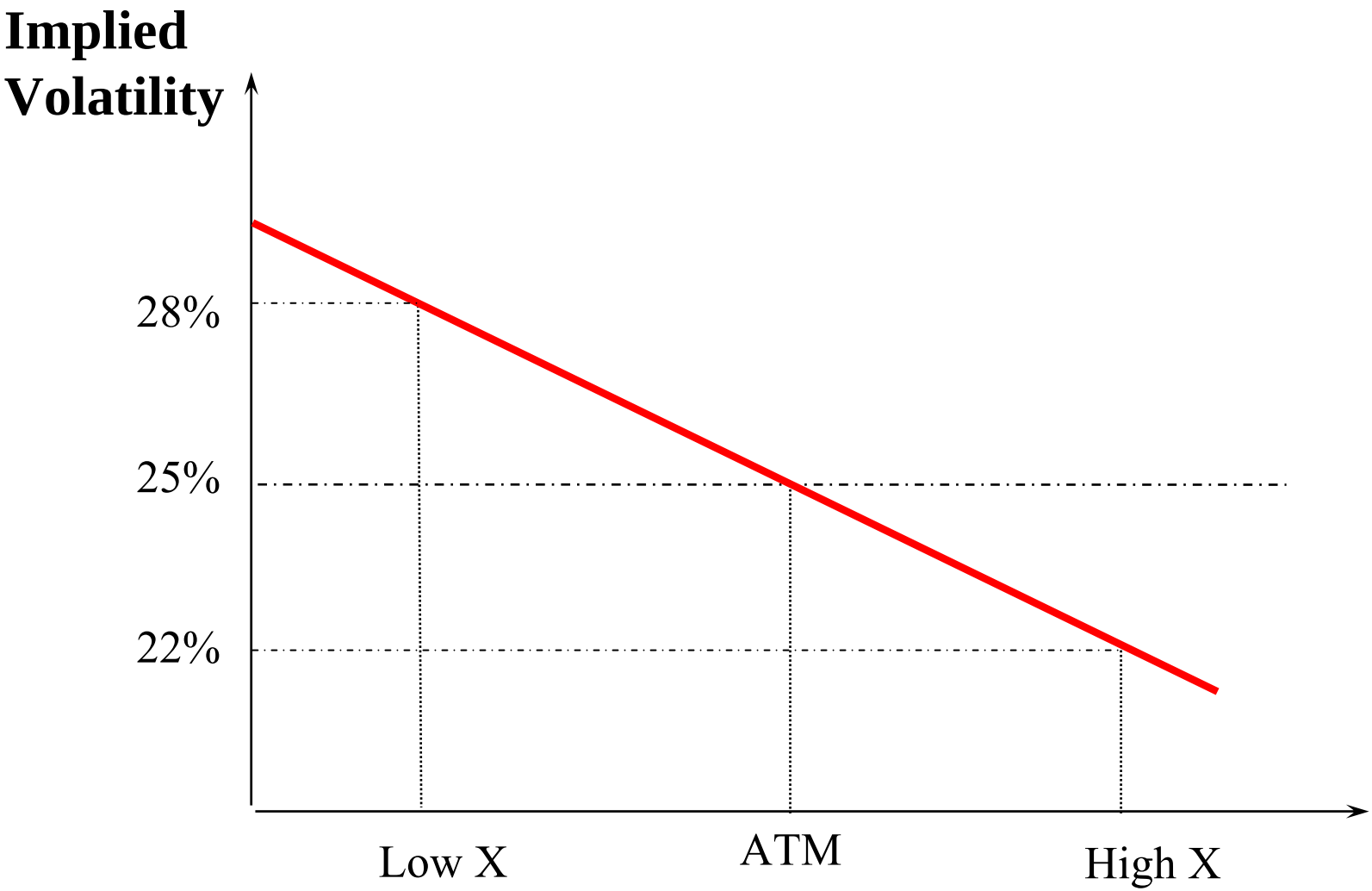
The 3 month-expiration options trade on different implied volatilities depending on the strike price.

How can the market simultaneously have different expectations for how the S&P 500 or IBM stock will fluctuate over the next three months?

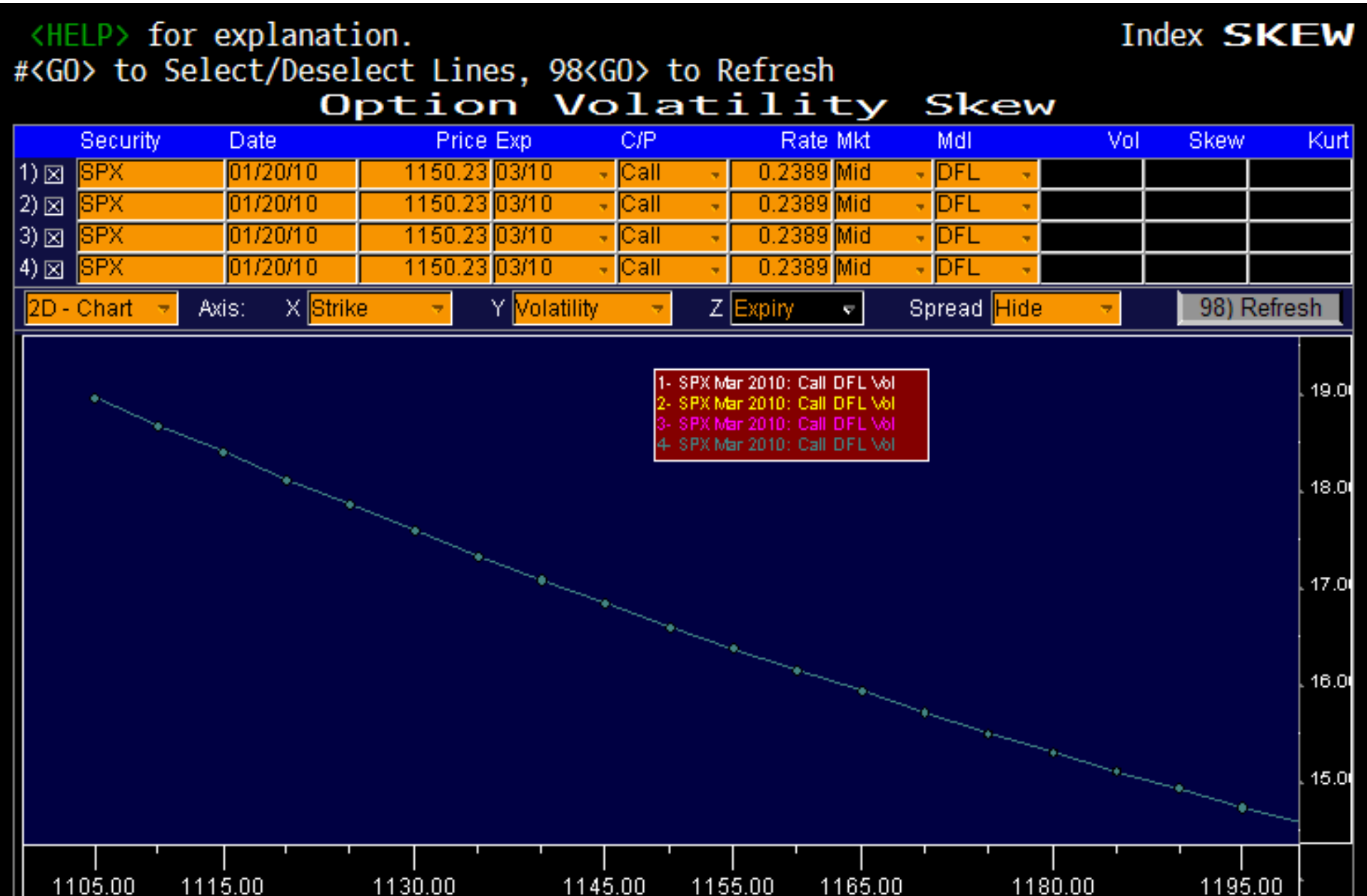
The plot of implied volatility versus strike price is called the "Volatility Smile" or "Vol Skew".

There is also a term structure of volatility.

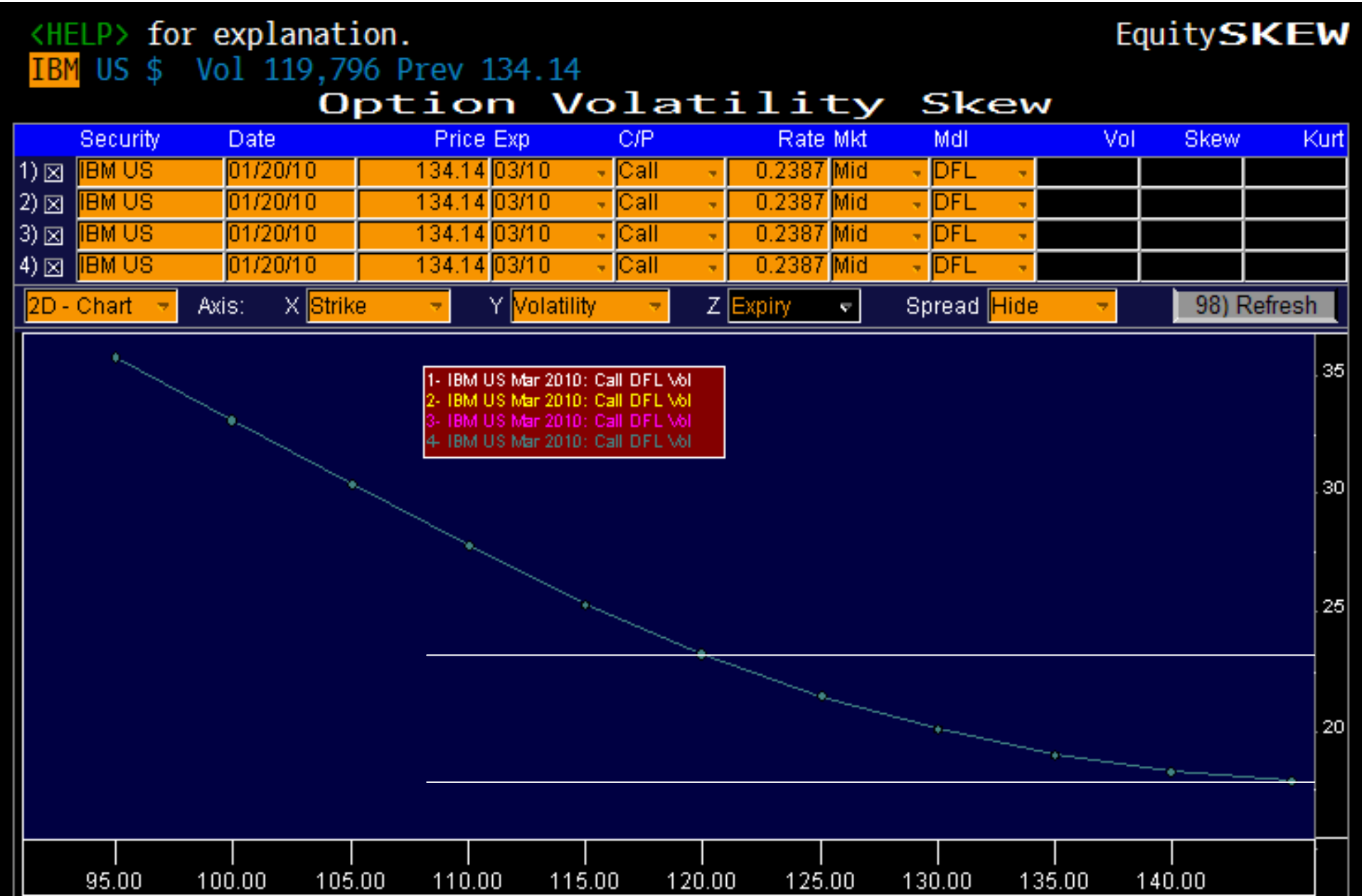
Typical Volatility Smile in Indexes



Actual Volatility Smile in S&P 500



Actual Volatility Smile in IBM



Option Trader's View of Skew

Why is there "skew"?

Demand and Supply

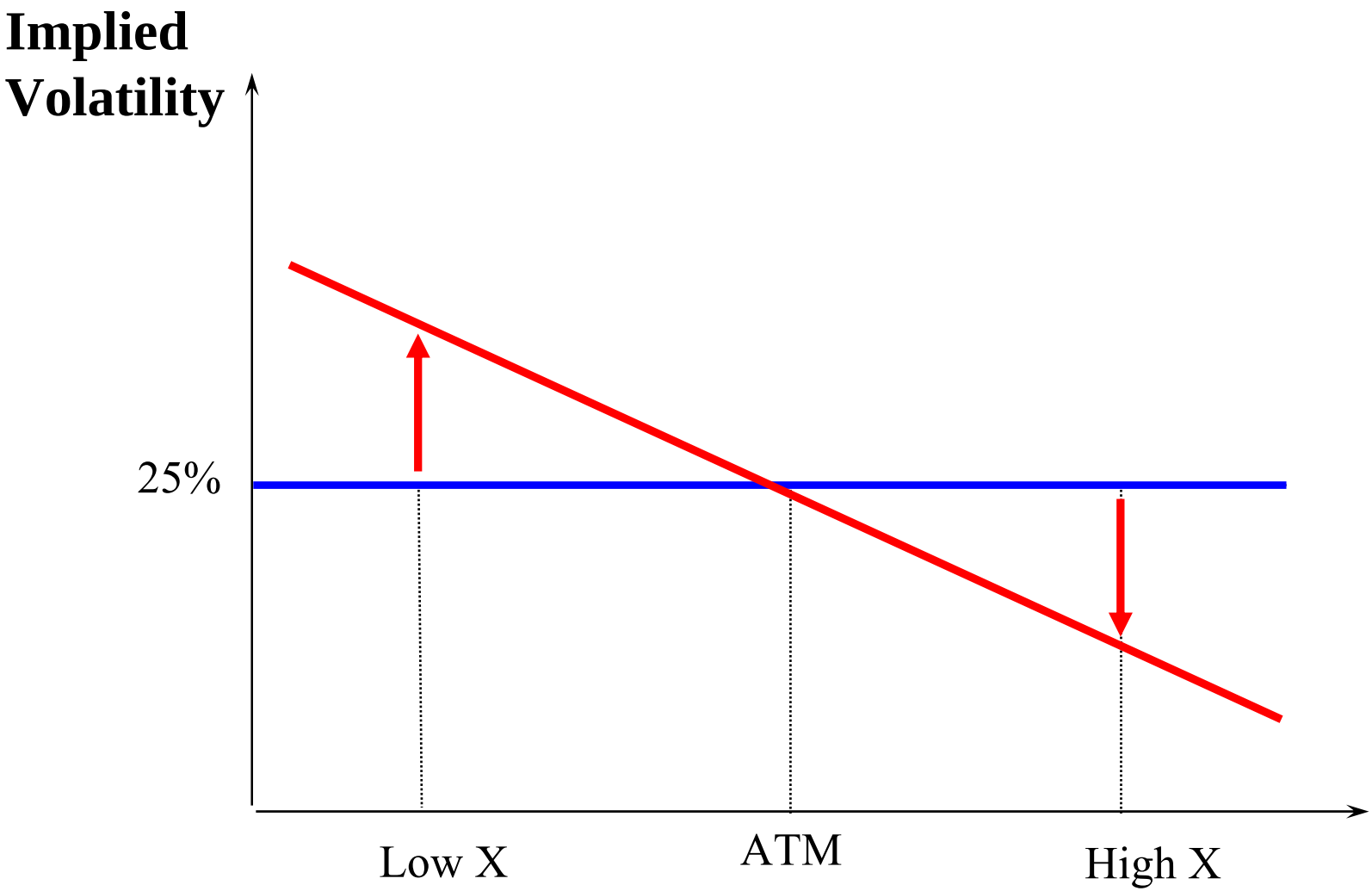
The world is "long stock"

so the world wants protection (buys OTM Puts)

and the world attempts to outperform the competition (sells OTM Calls – yield enhancement, an income strategy)

and sometimes does both simultaneously (Collars)

As a result of buying OTM Puts and selling OTM Calls, . . .



The Market Is Not Stupid

The market remembers:

Stocks tend to fall faster than they rise.

(This is an empirical statement.)

Even if this isn't true (but people believe it), explains the interest in purchasing Puts (relative to Calls).

Puts over Calls
(holding something constant: 20% OTM, 10Δ,…)

Why is there an Equity Skew?

(Low-Strike) Puts [Protective Put Strategy] are bid.

(High-Strike) Calls [Covered Calls, Over-Writes, Buy-Writes Strategy] are offered.

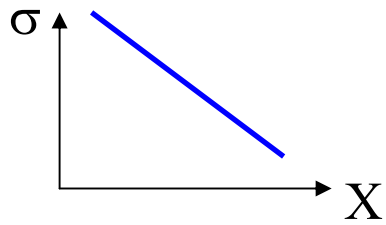
Just the result of people buying and selling options.

What about other product areas?
Is the world long Yen?

Different Skews for Different Products

Equity Skews typically have negative slope:

Investment Skew

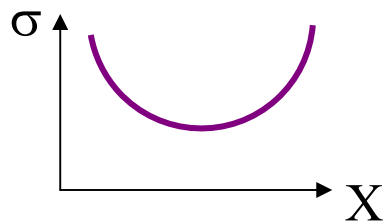


Commodities (which “crash” up) have positive skews:

Demand Skew

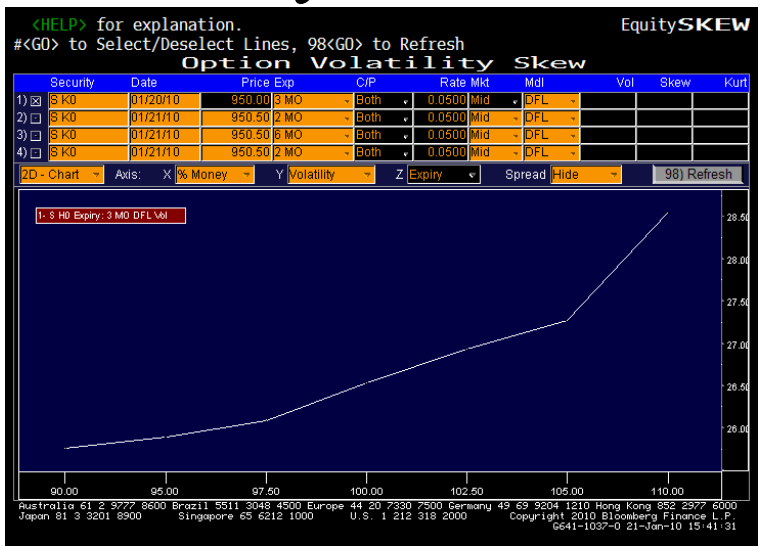
Foreign Exchange has a more symmetric skew:

Balanced Skew

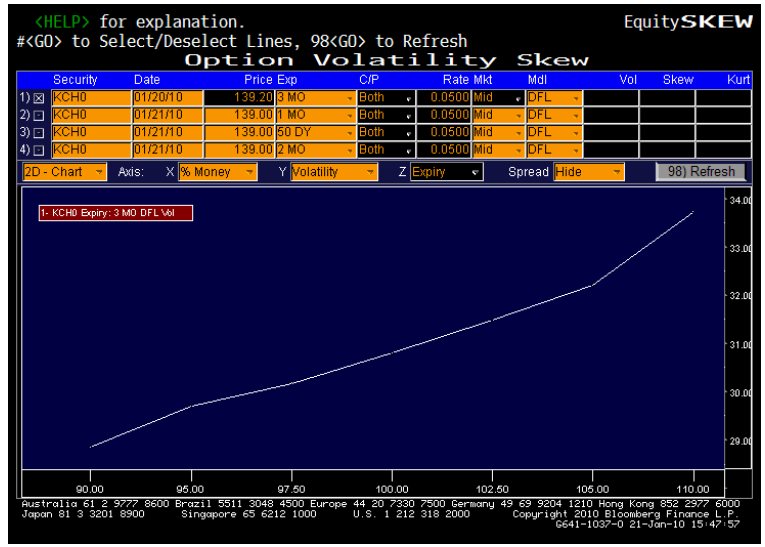


Actual Volatility Smiles in Commodities

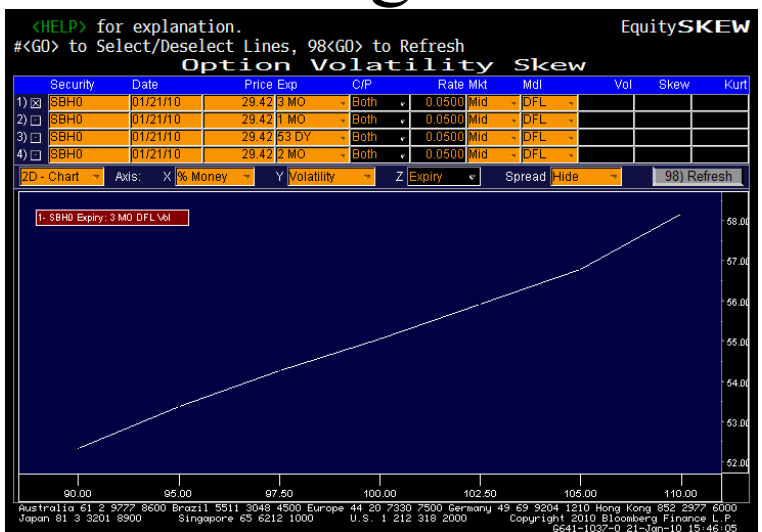
Soybeans



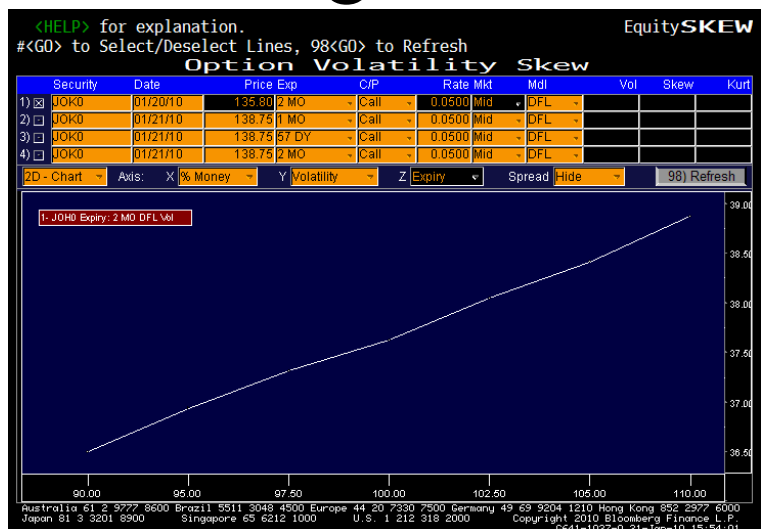
Coffee



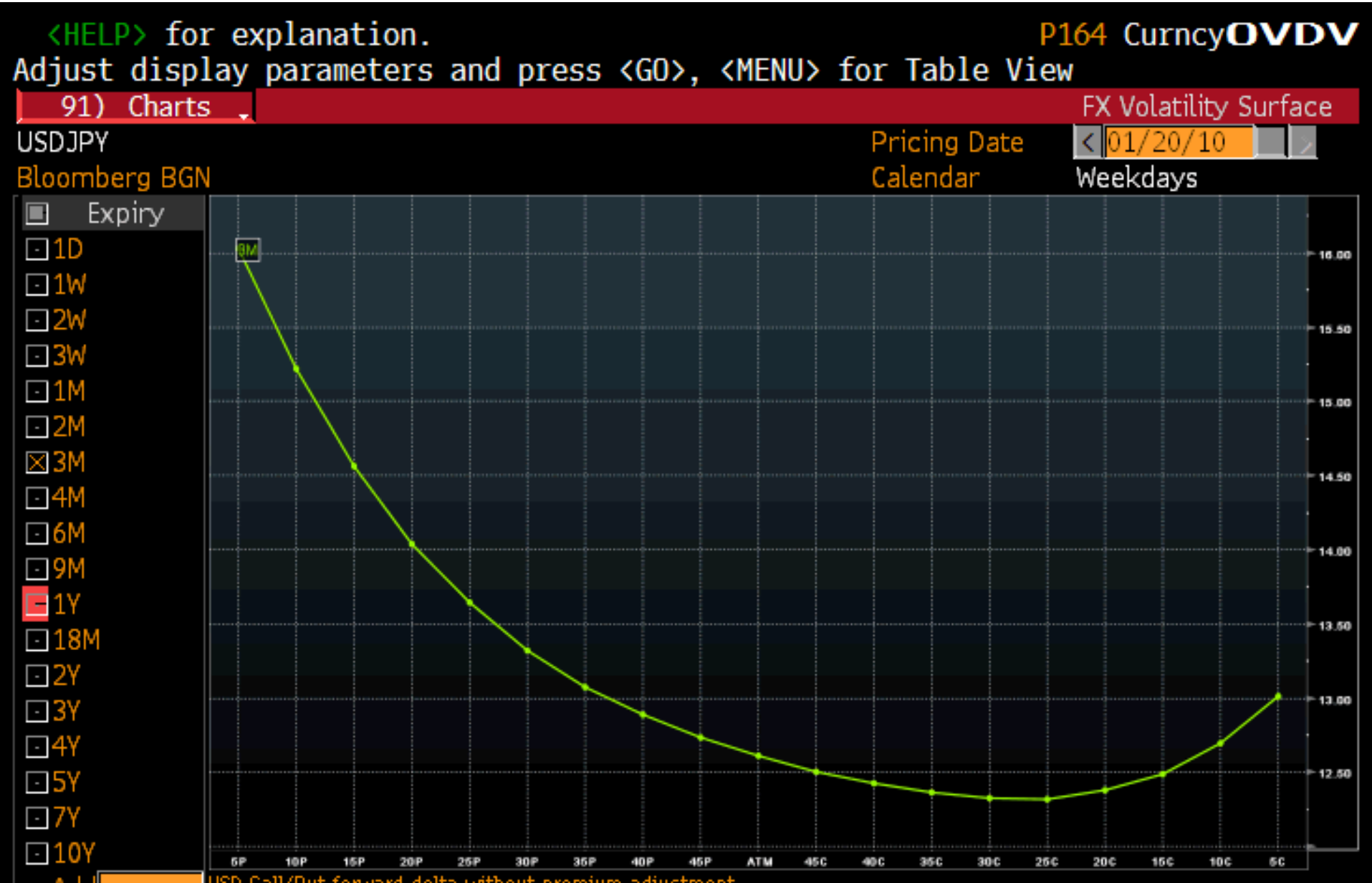
Sugar



Orange Juice



Actual Volatility Smile in FX (USD|JPY)



Skew: What is it?

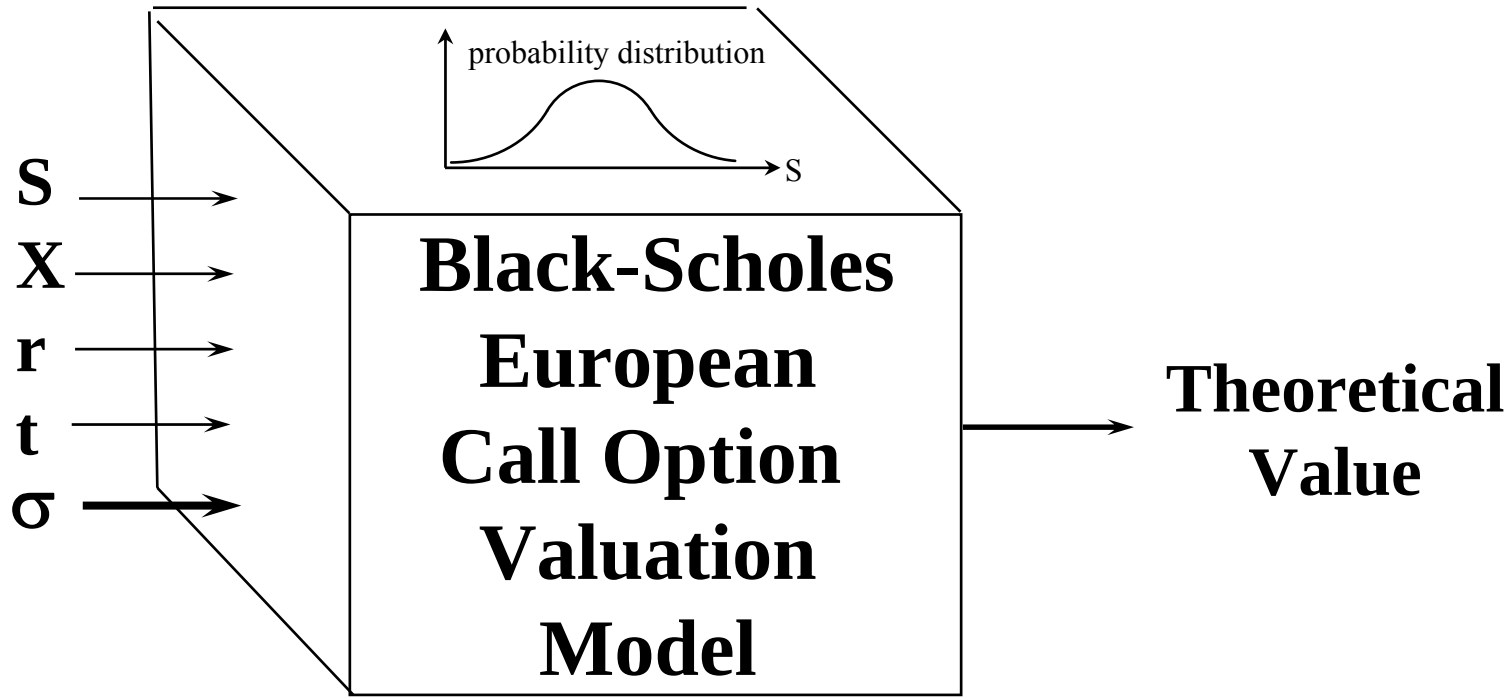
"Skew" means different things to different people:

To a Financial Engineer:

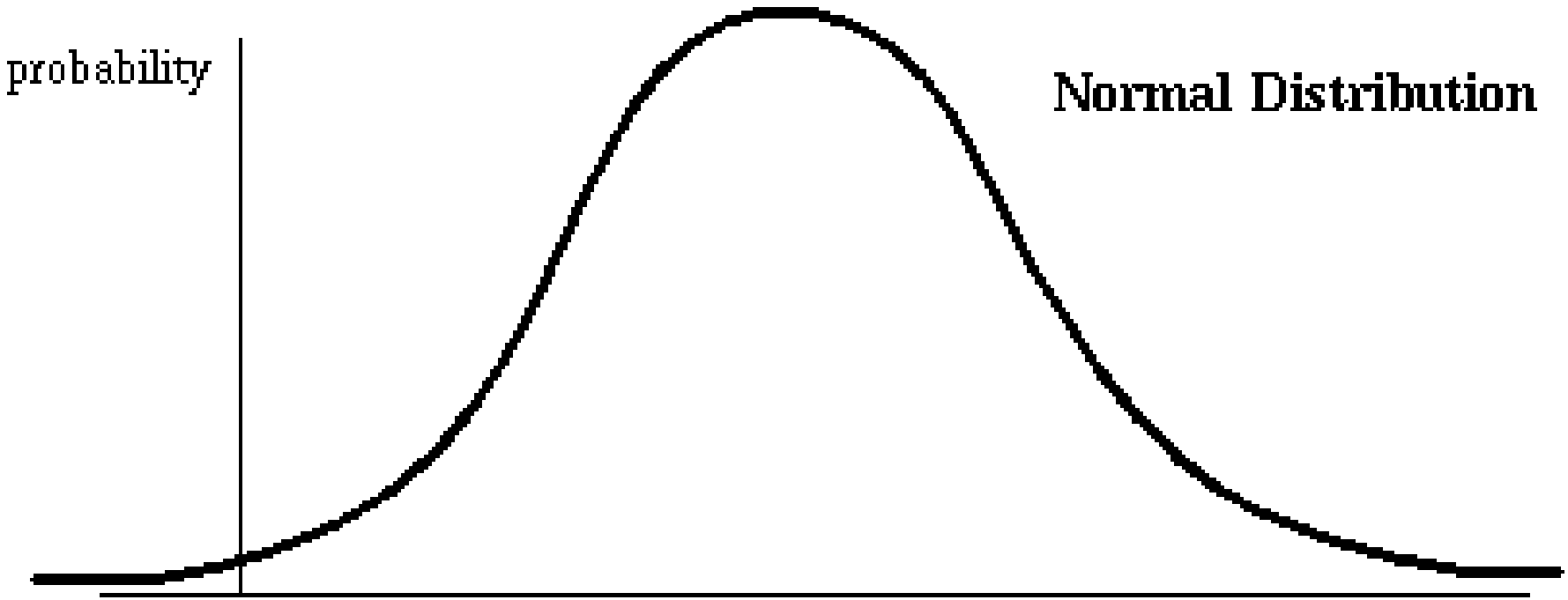
the market phenomenon which results from using a model which, like all models, is not correct.

In other words, the model assumes a probability distribution (more precisely, a probability density function) for the future price distribution of the underlying (lognormal) – which is "wrong"!

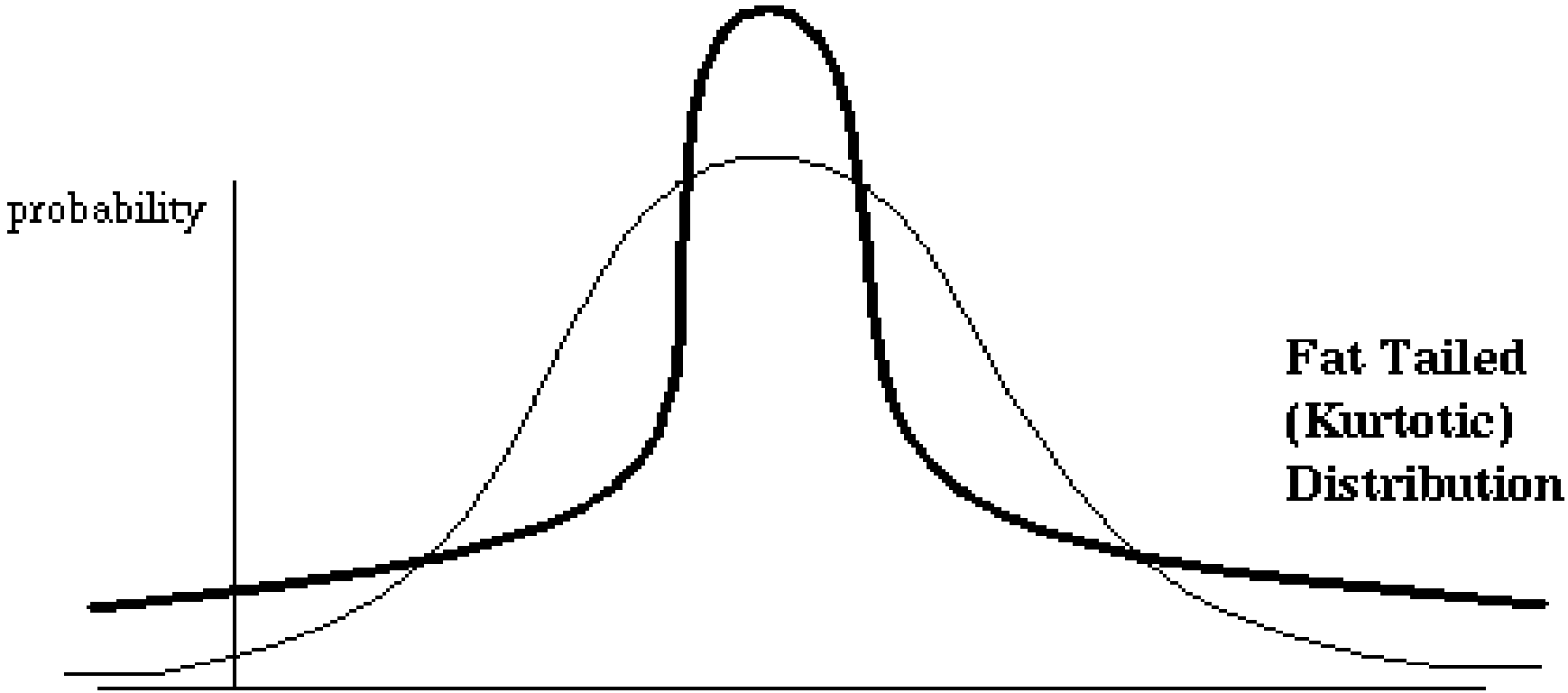
What's in the Black(-Scholes) Box?



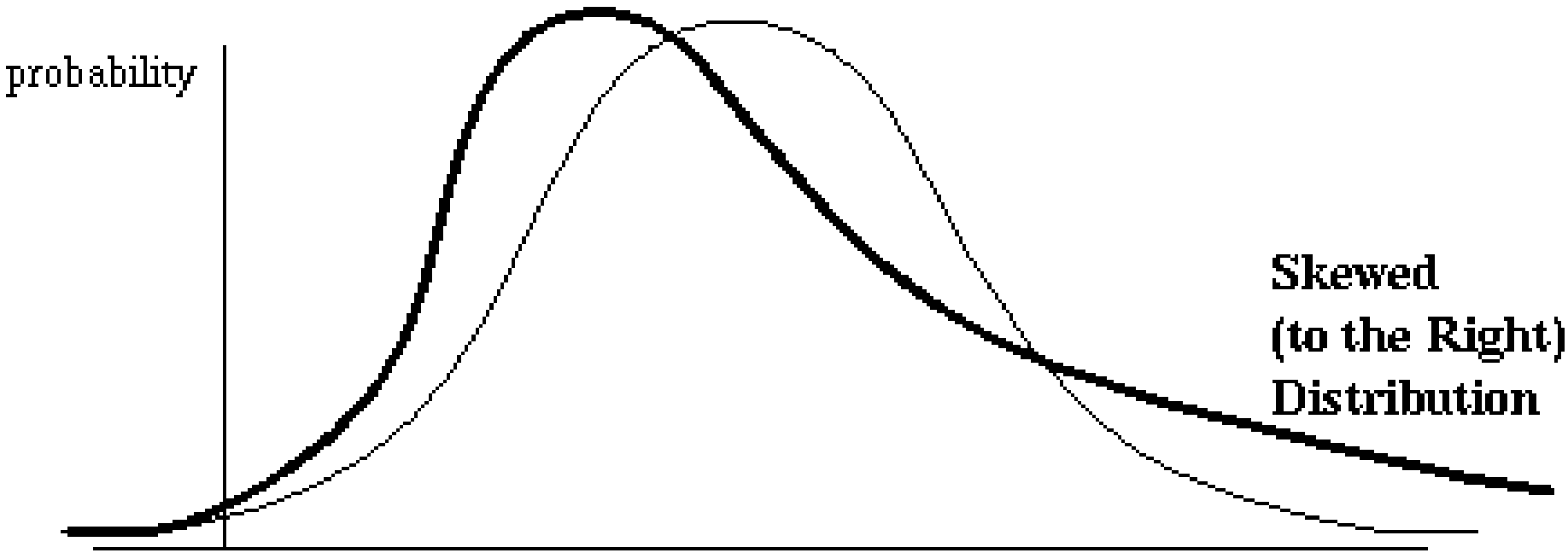
Baseline Distribution

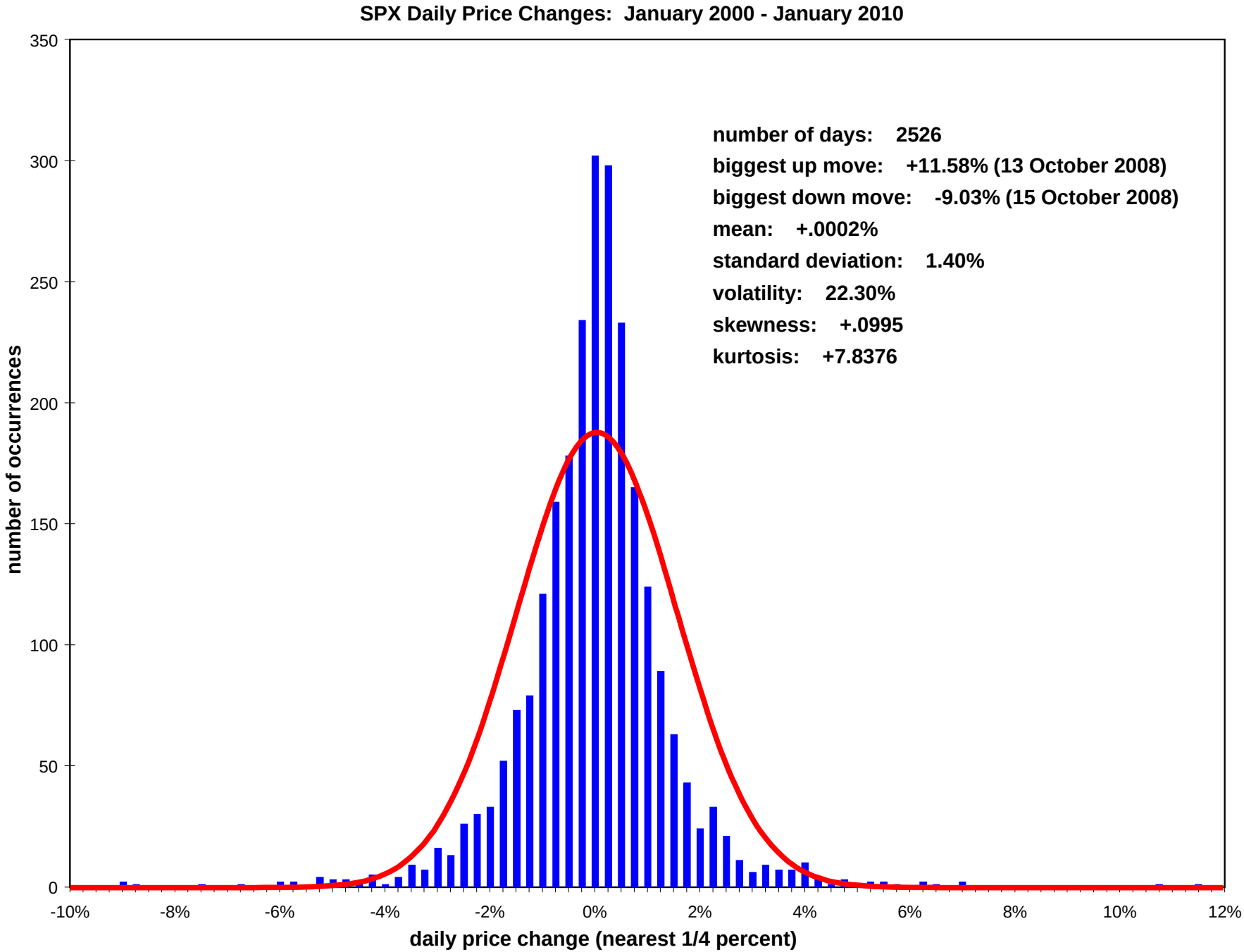


Kurtosis

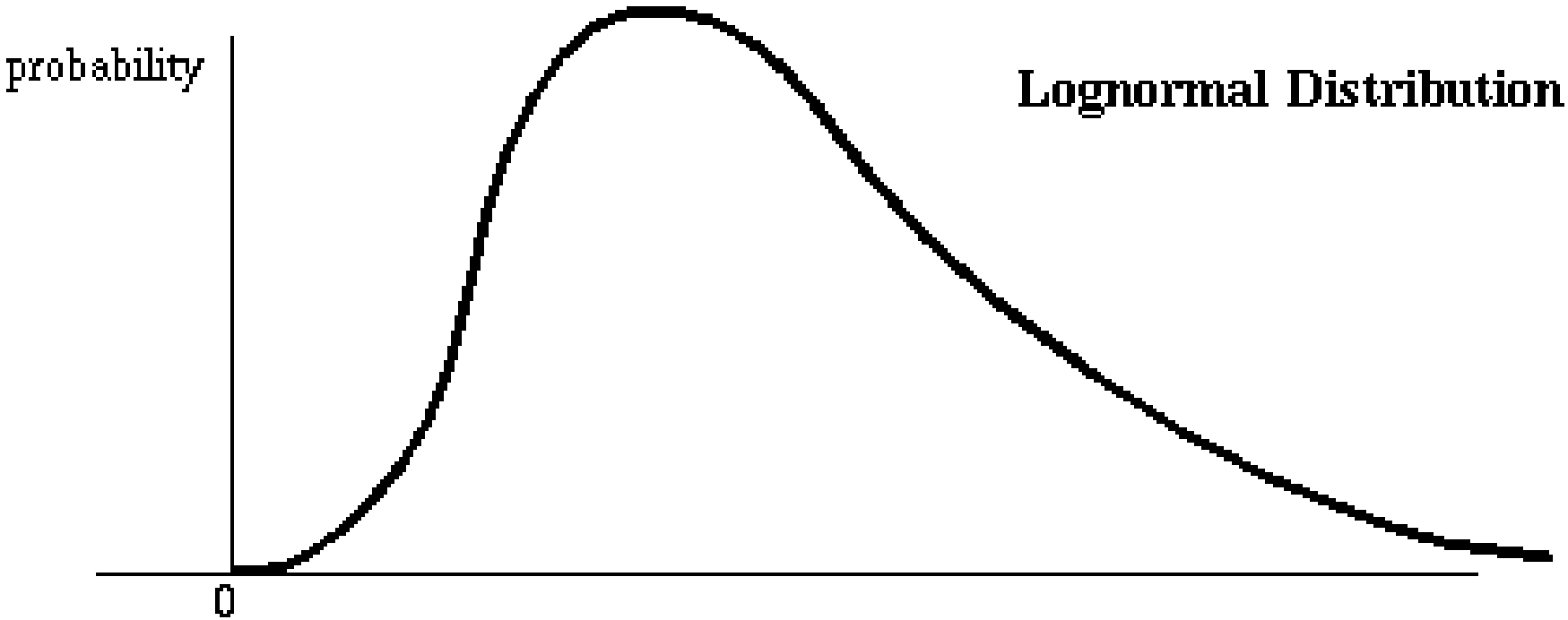


Skew




SPX Daily Price Changes: January 2000 - January 2010
number of days: 2526
biggest up move: +11.58% (13 October 2008)
biggest down move: -9.03% (15 October 2008)
mean: +.0002%
standard deviation: 1.40%
volatility: 22.30%
skewness: +.0995
kurtosis: +7.8376
number of occurrences
0
50
100
150
200
250
300
350
-10%
-8%
-6%
-4%
-2%
0%
2%
4%
6%
8%
10%
12%
daily price change (nearest 1/4 percent)

Prices are assumed to follow a Lognormal Distribution



So, really, “skewed” relative to this . . .

Skew: What is it?

"Skew" means different things to different people:

To a Risk Manager:

the relationship between movements in the underlying asset price and implied volatilities

What will happen to implied volatility as the underlying price goes down or goes up (and possibly goes down or up slowly versus goes down or up quickly)?

Market implied volatility is a function of spot price: $\sigma(S)$

Modelling the Skew: What Do You Believe?

Skew is "real" in that it impacts your mark-to-market.

What this means is that in order to "risk manage"
a portfolio containing options, you have to
take the skew into account . . . and, ultimately,
to be systematic, consistent, robust, . . .
you will want to formally "model the skew."

Volatility by Moneyness (ATM, 10% OTM vol constant)

Volatility by Strike (Sticky Strike: vol by strike constant)

Volatility by Delta (Sticky Delta: vol by Delta constant)

And How Do You Manage It?

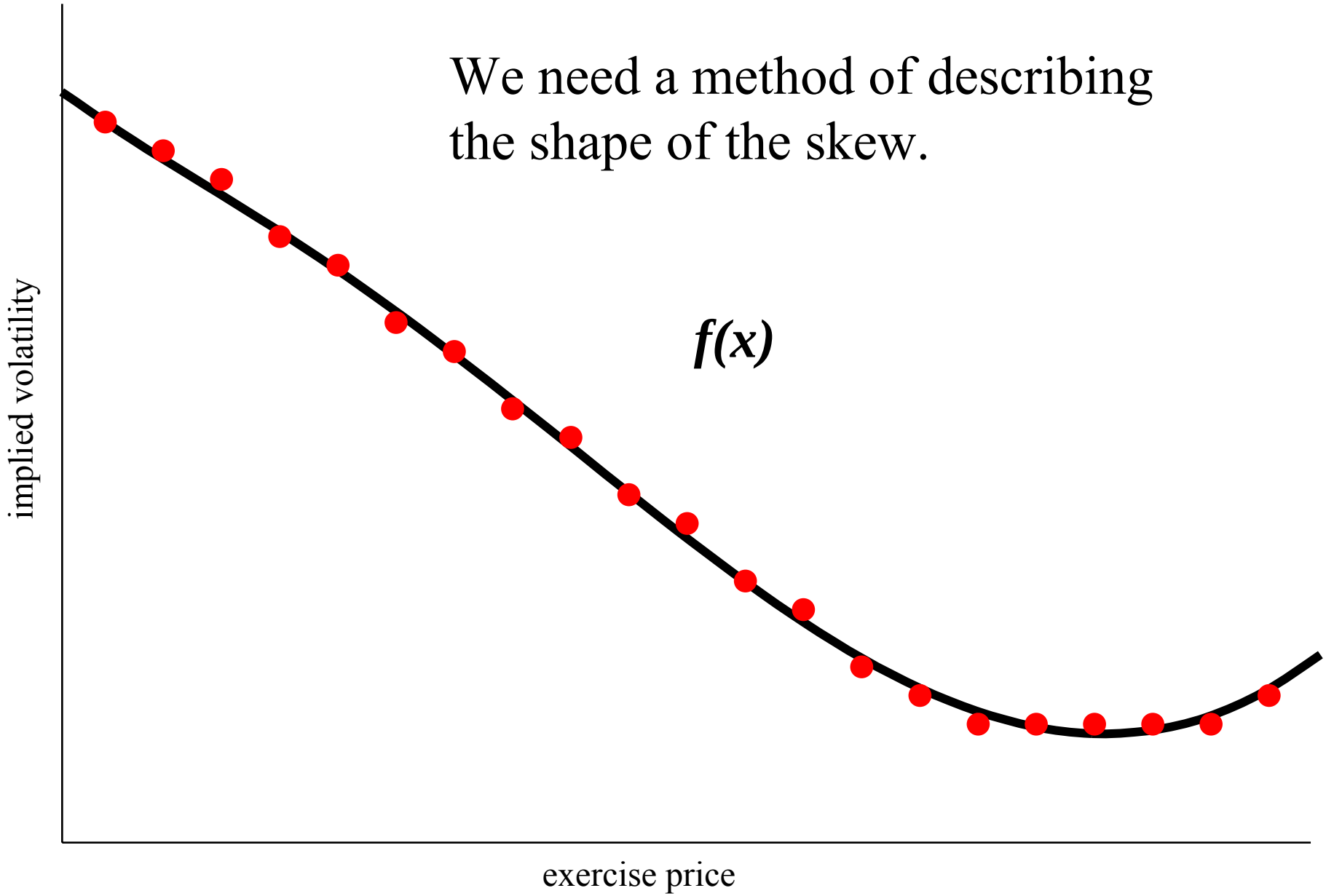
You can't just ignore skew; you have to "model" it.

Skew can and will change.

Based on your view on skew,
some trades may look attractive
(or more attractive than others).

Shelly: How to model skew?

What is a skew trade?

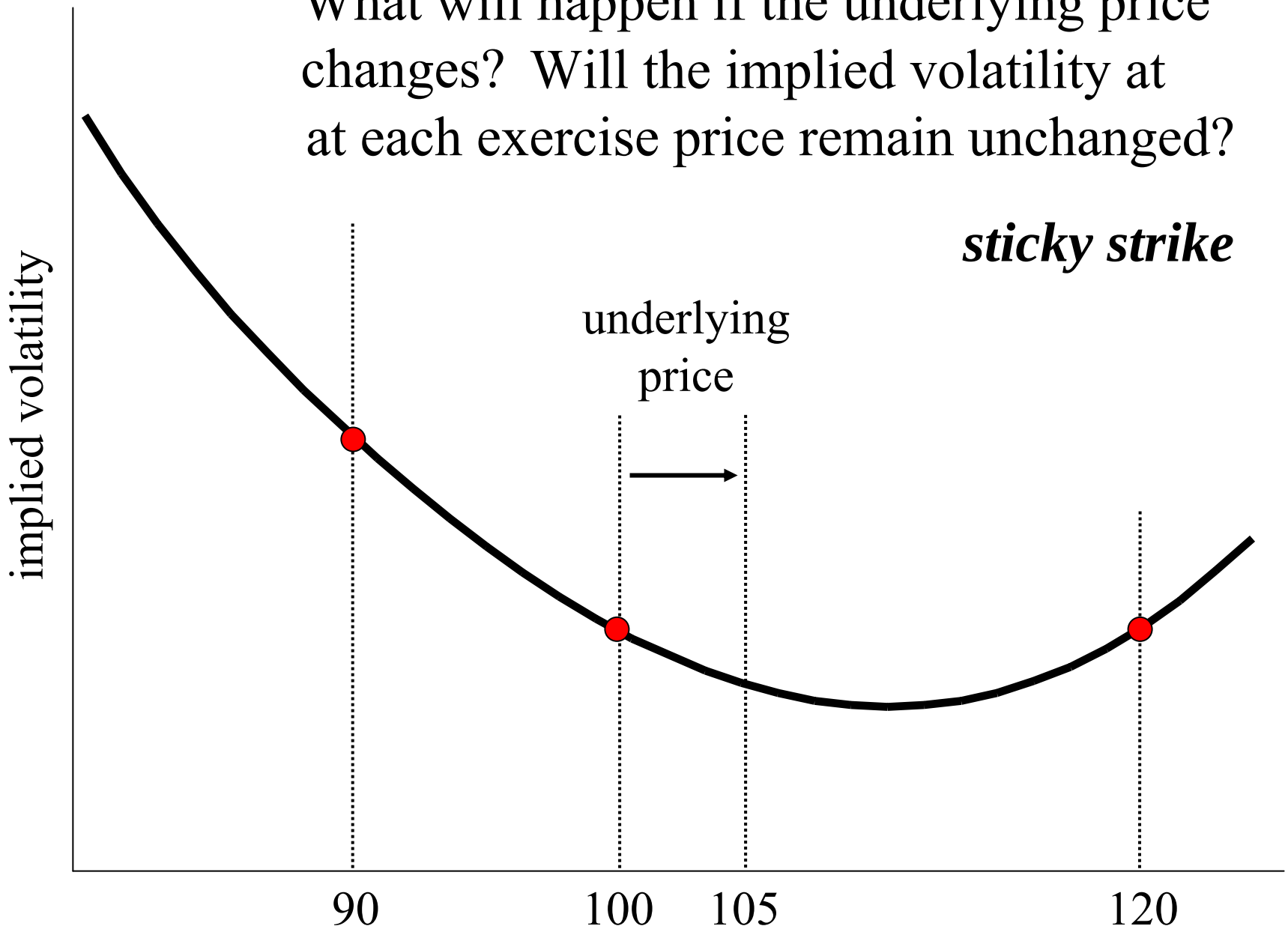

We need a method of describing the shape of the skew.
f(x)
implied volatility
exercise price

In order to model option values and determine the risk of a position we need to know how changes in market conditions will affect…

the location of the skew

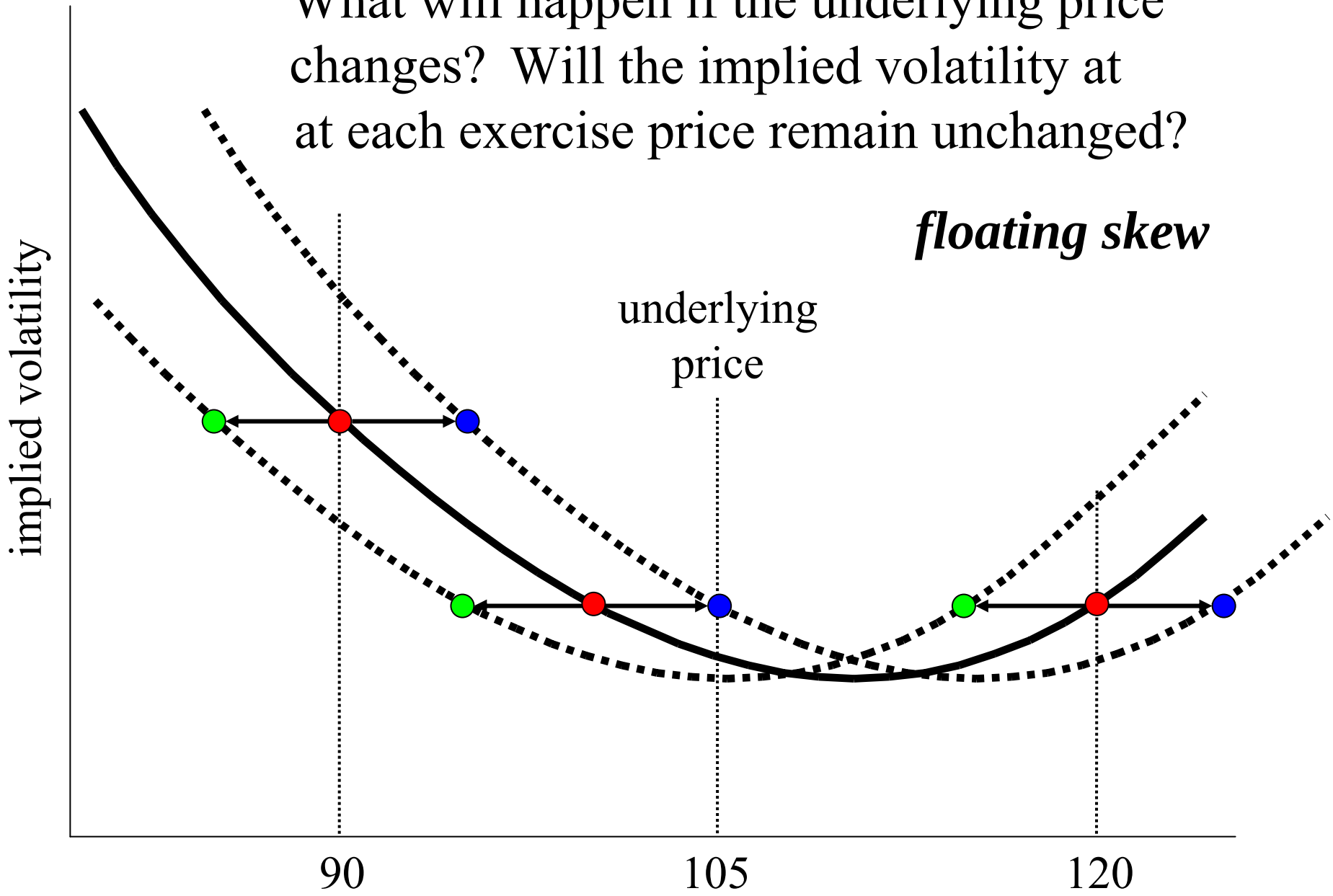
the shape of the skew

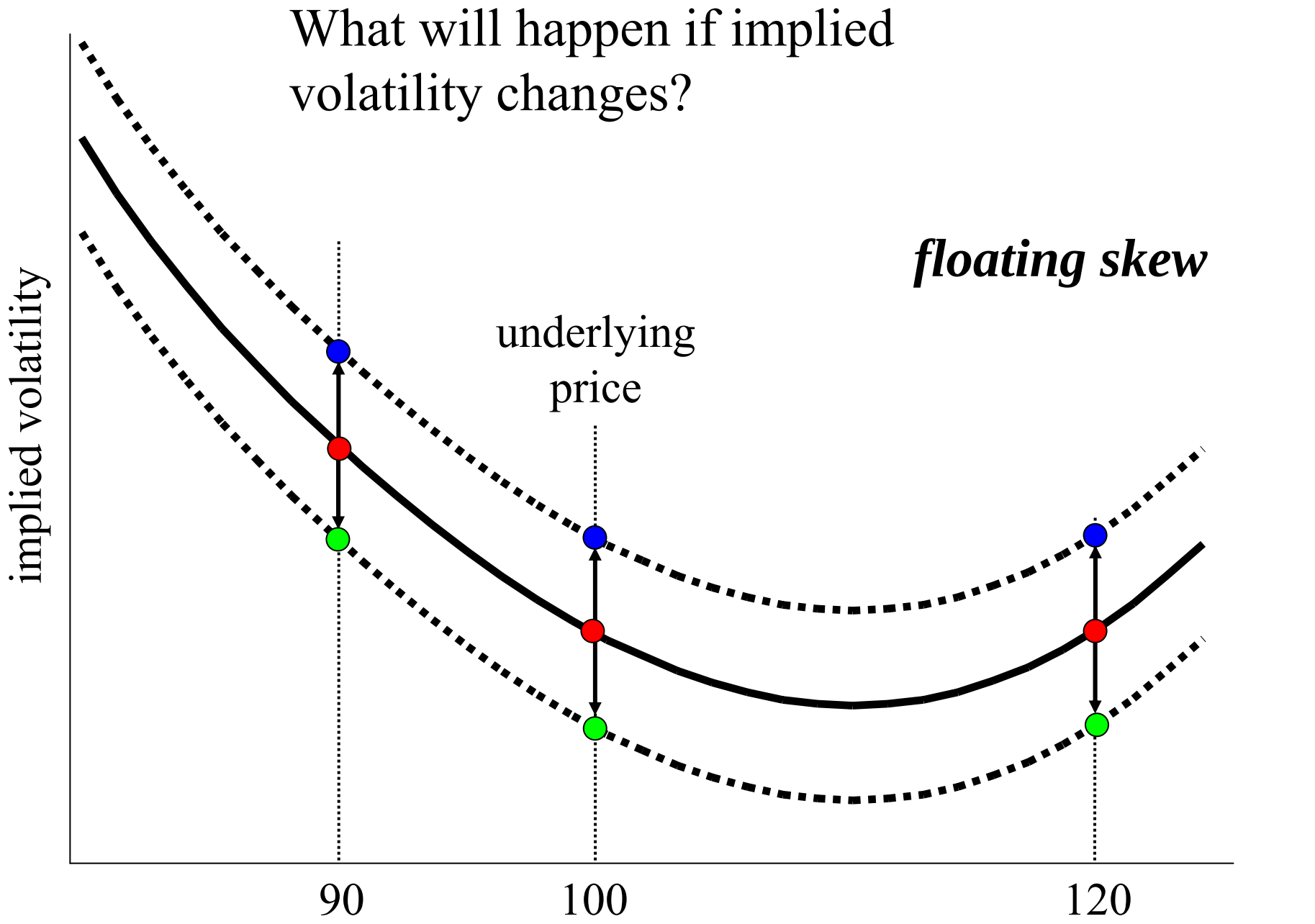
What will happen if the underlying price changes? Will the implied volatility at at each exercise price remain unchanged?

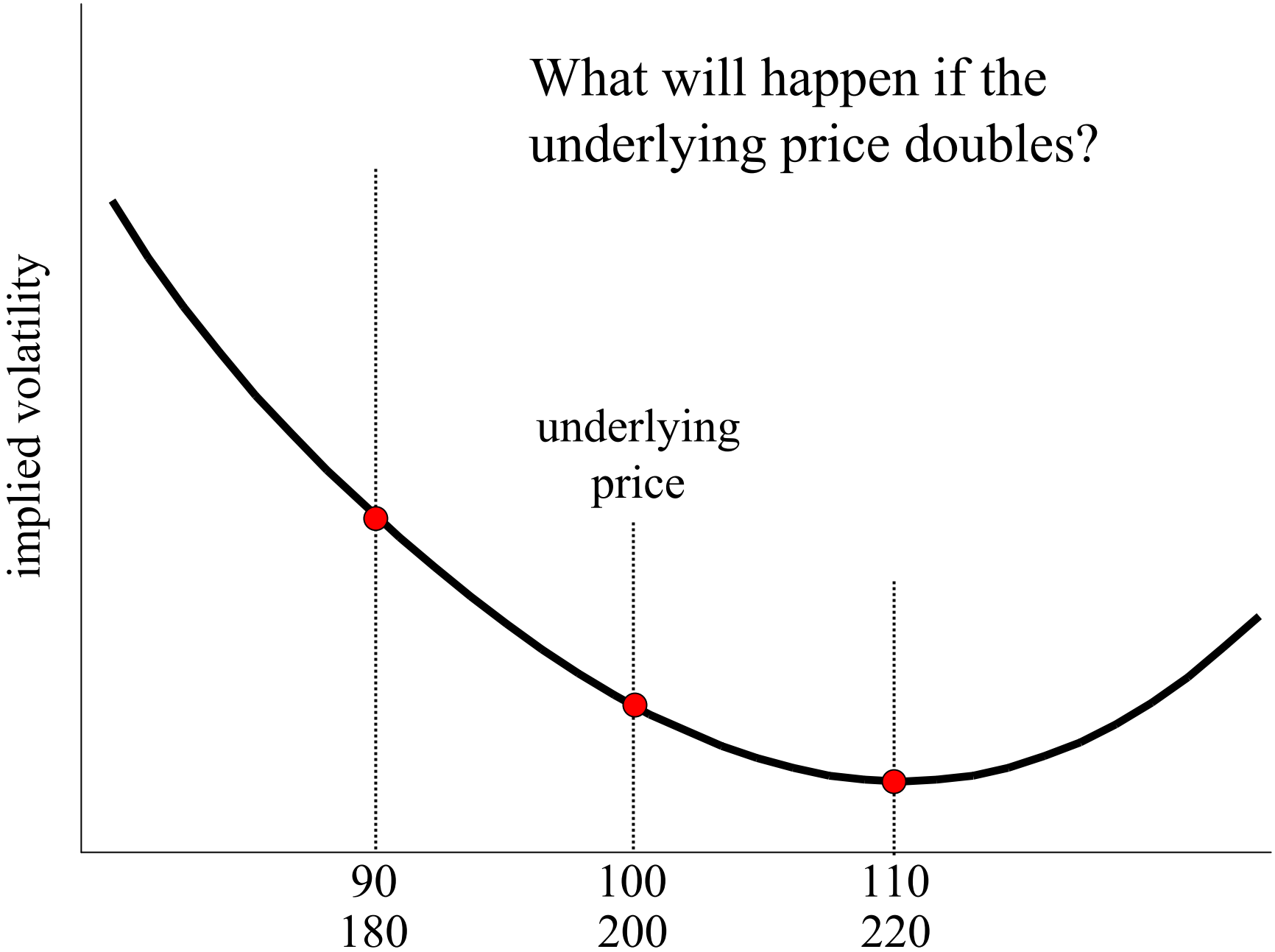


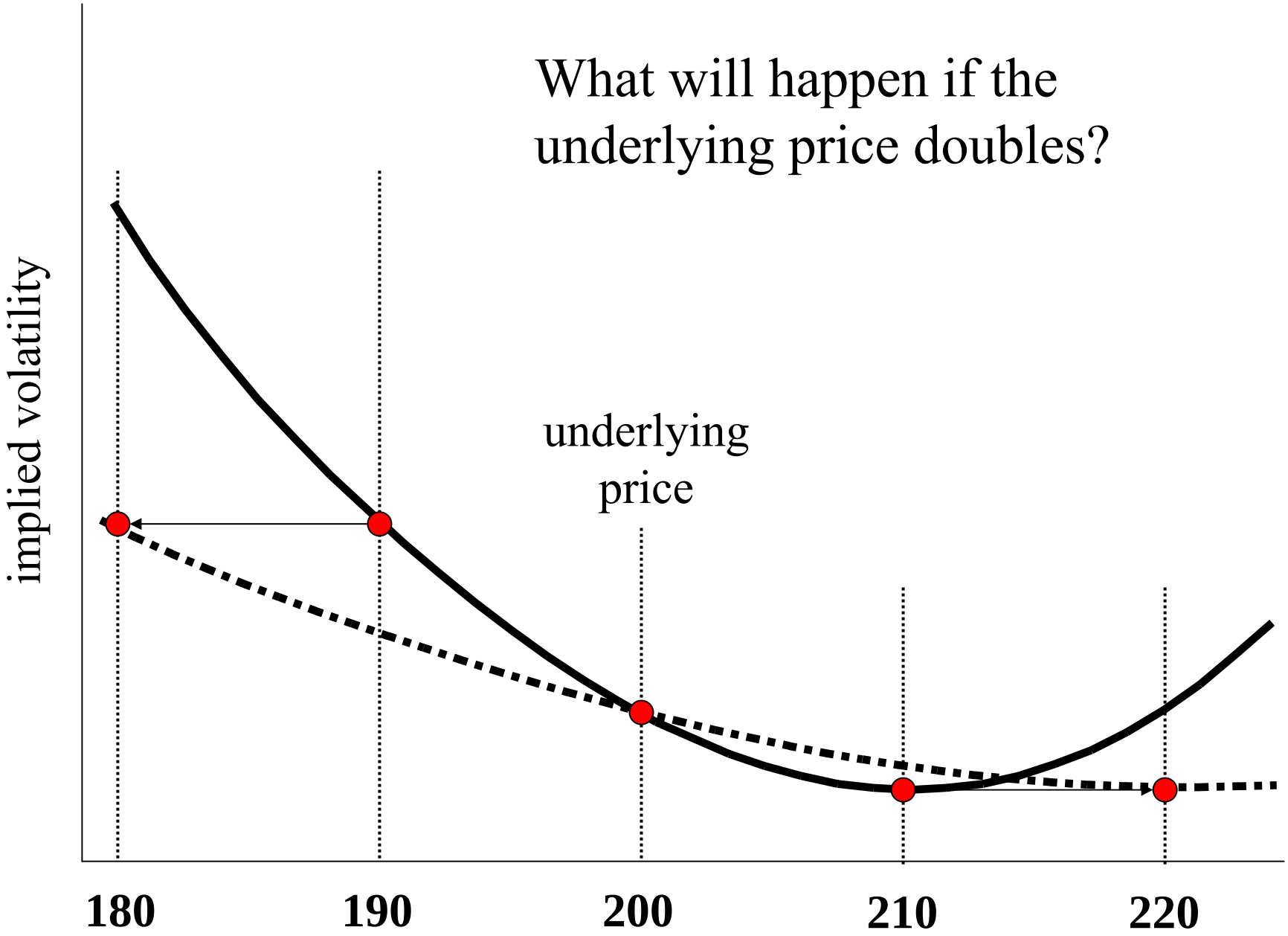
What will happen if the underlying price changes? Will the implied volatility at at each exercise price remain unchanged?

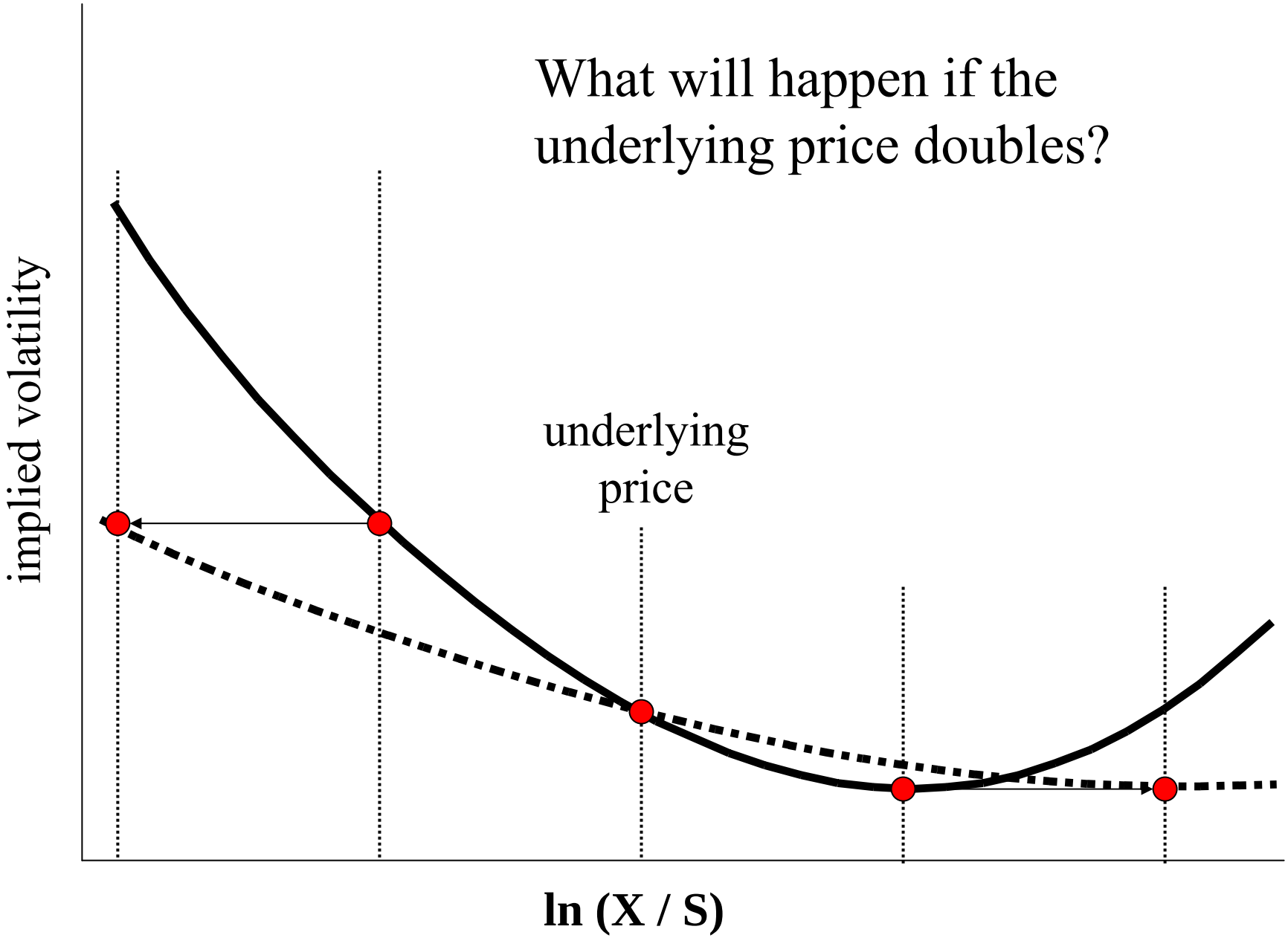
floating skew




What will happen if implied volatility changes?
floating skew
implied volatility
underlying price
90
100
120

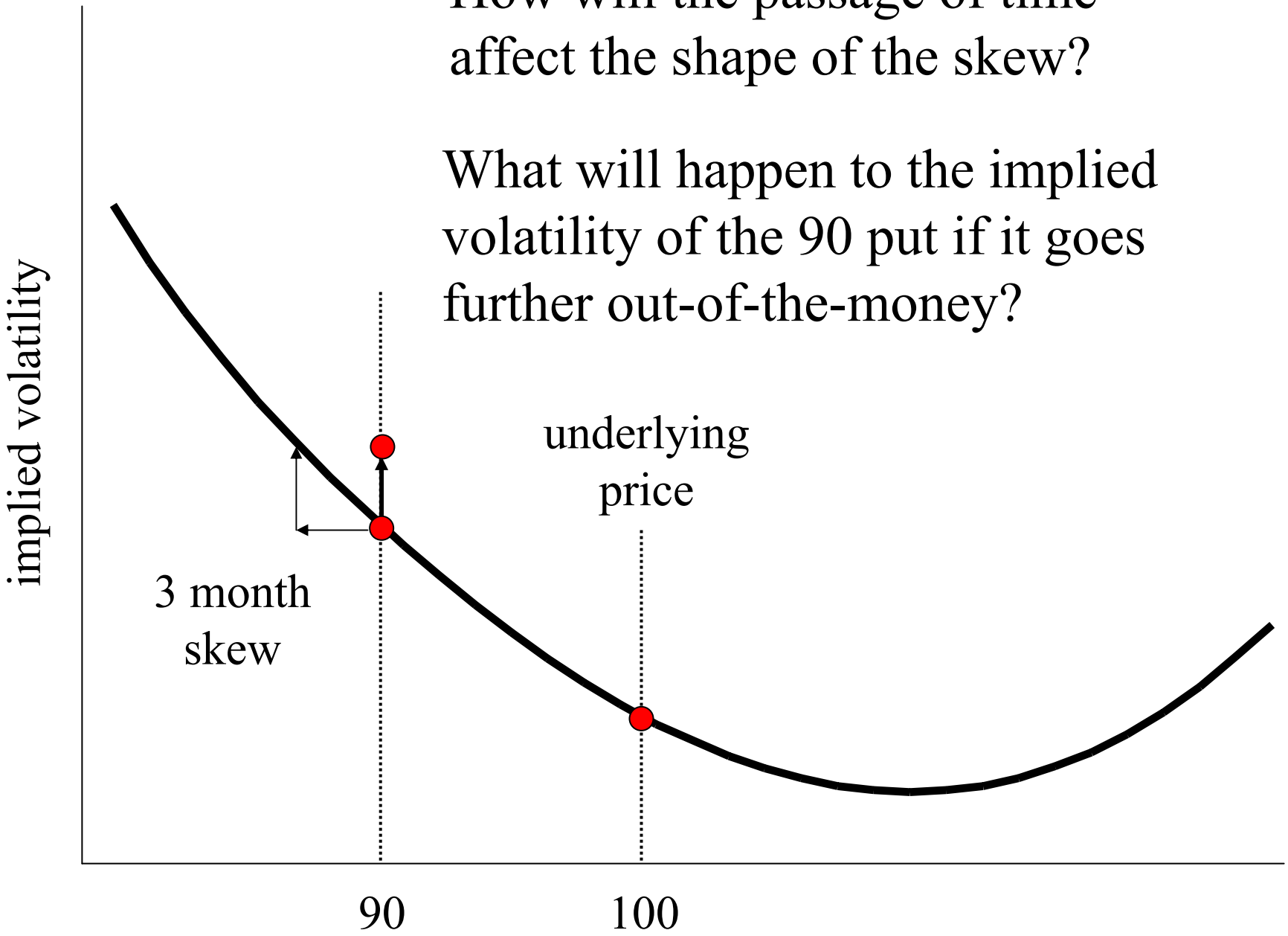

What will happen if the
underlying price doubles?
implied volatility
underlying
price
90
180
100
200
110
220


What will happen if the underlying price doubles?
implied volatility
underlying price
180
190
200
210
220


What will happen if the underlying price doubles?
implied volatility
underlying price
ln (X / S)

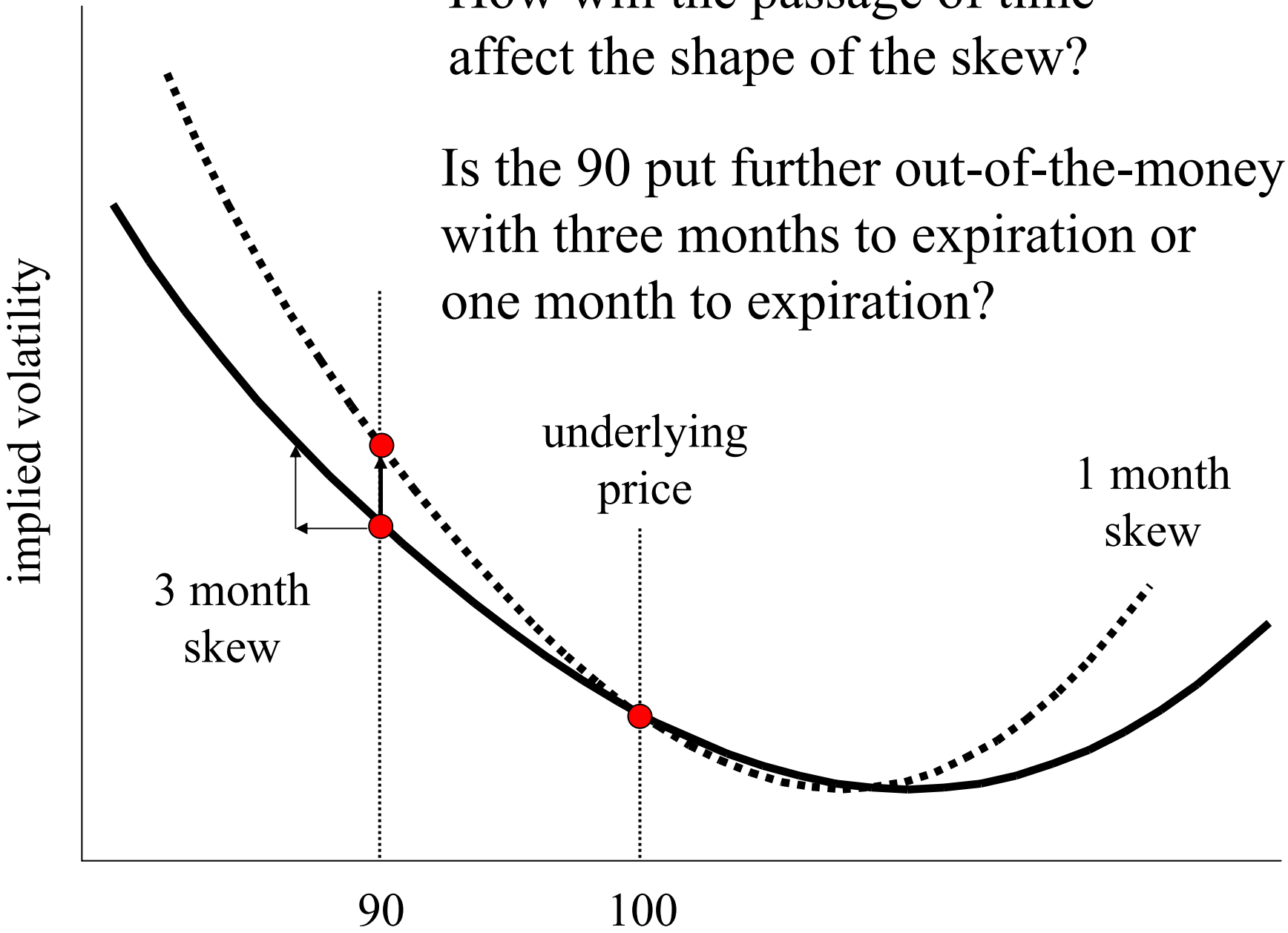
How will the passage of time affect the shape of the skew?

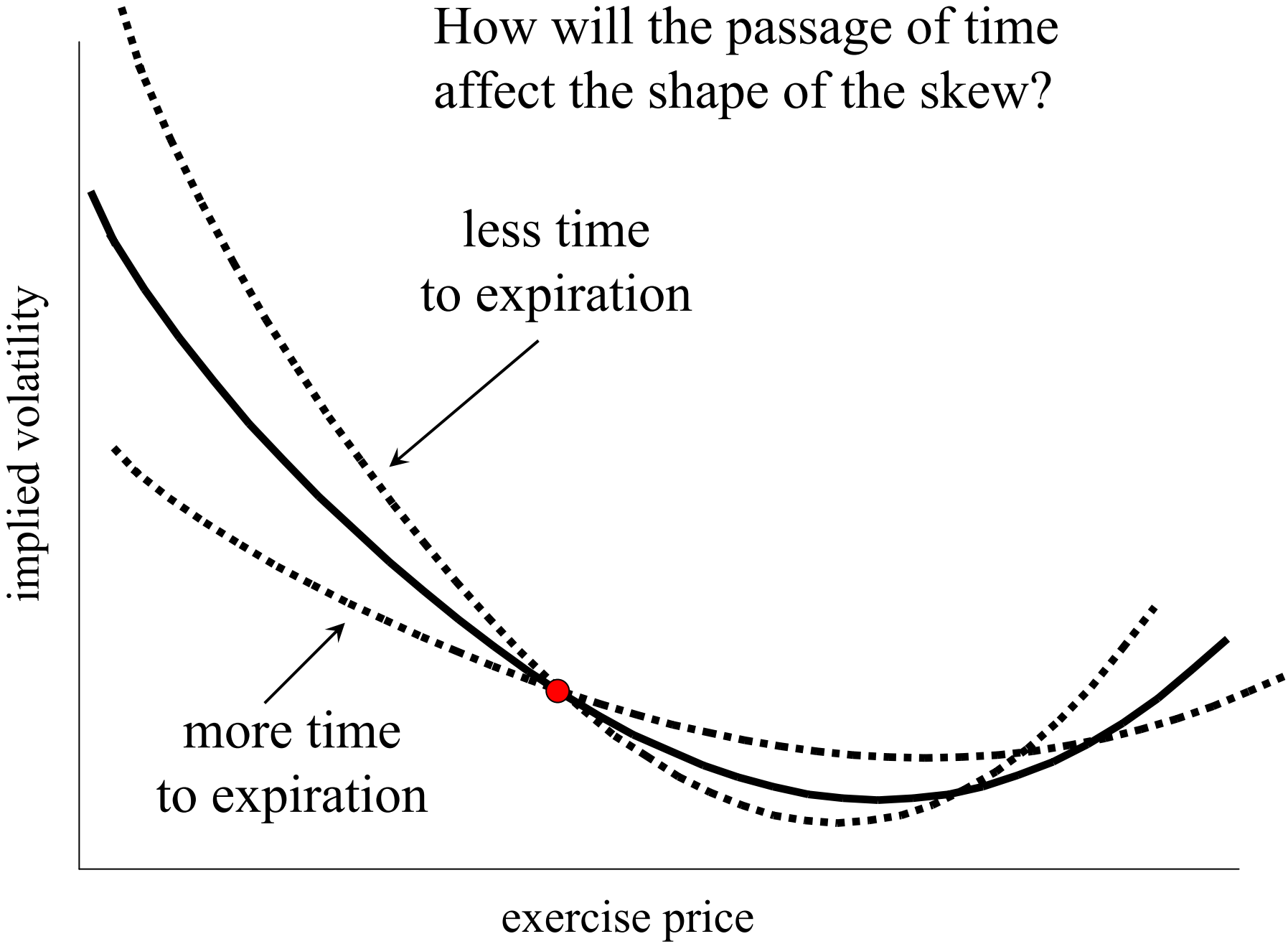
What will happen to the implied volatility of the 90 put if it goes further out-of-the-money?

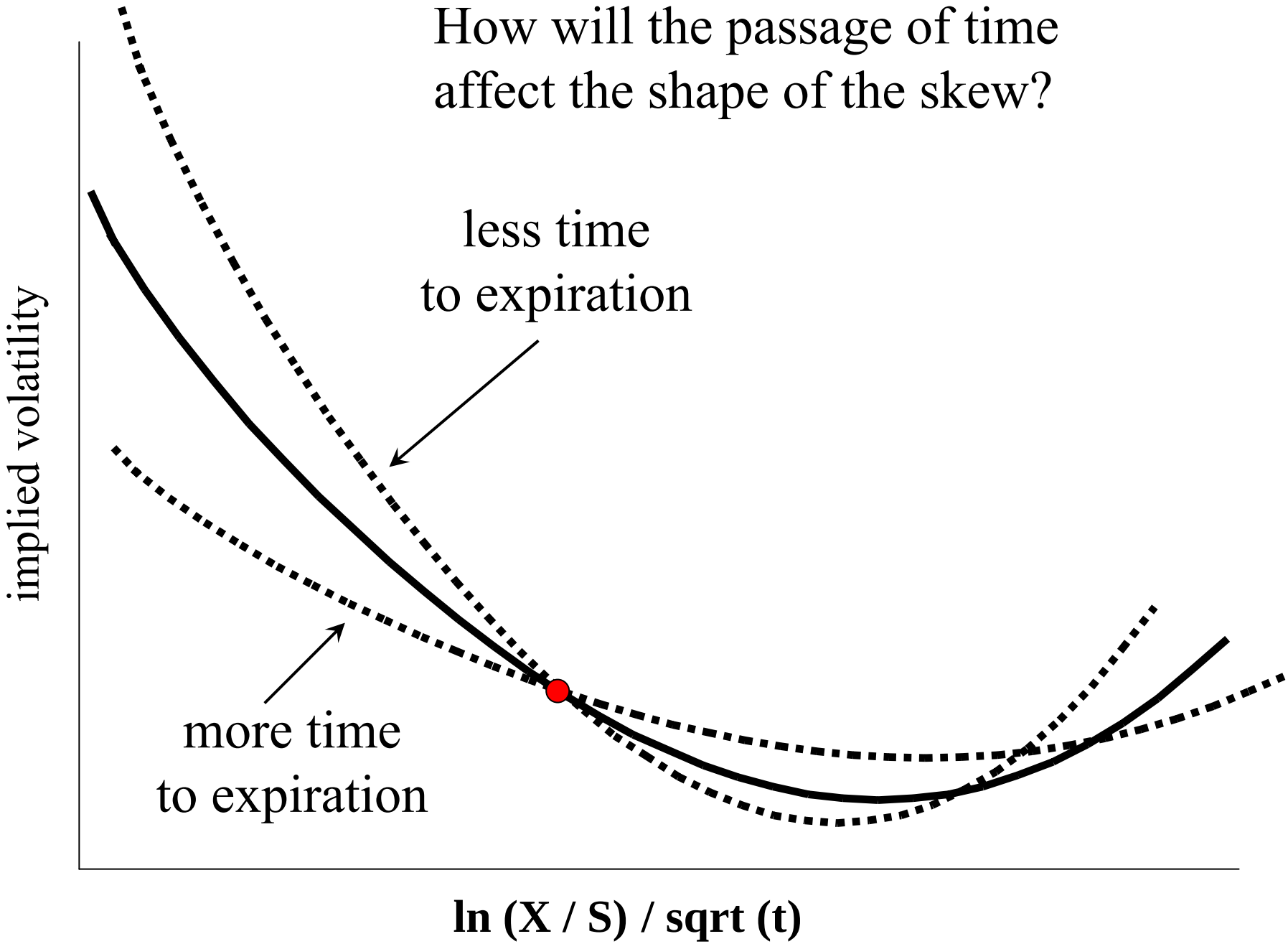


How will the passage of time affect the shape of the skew?

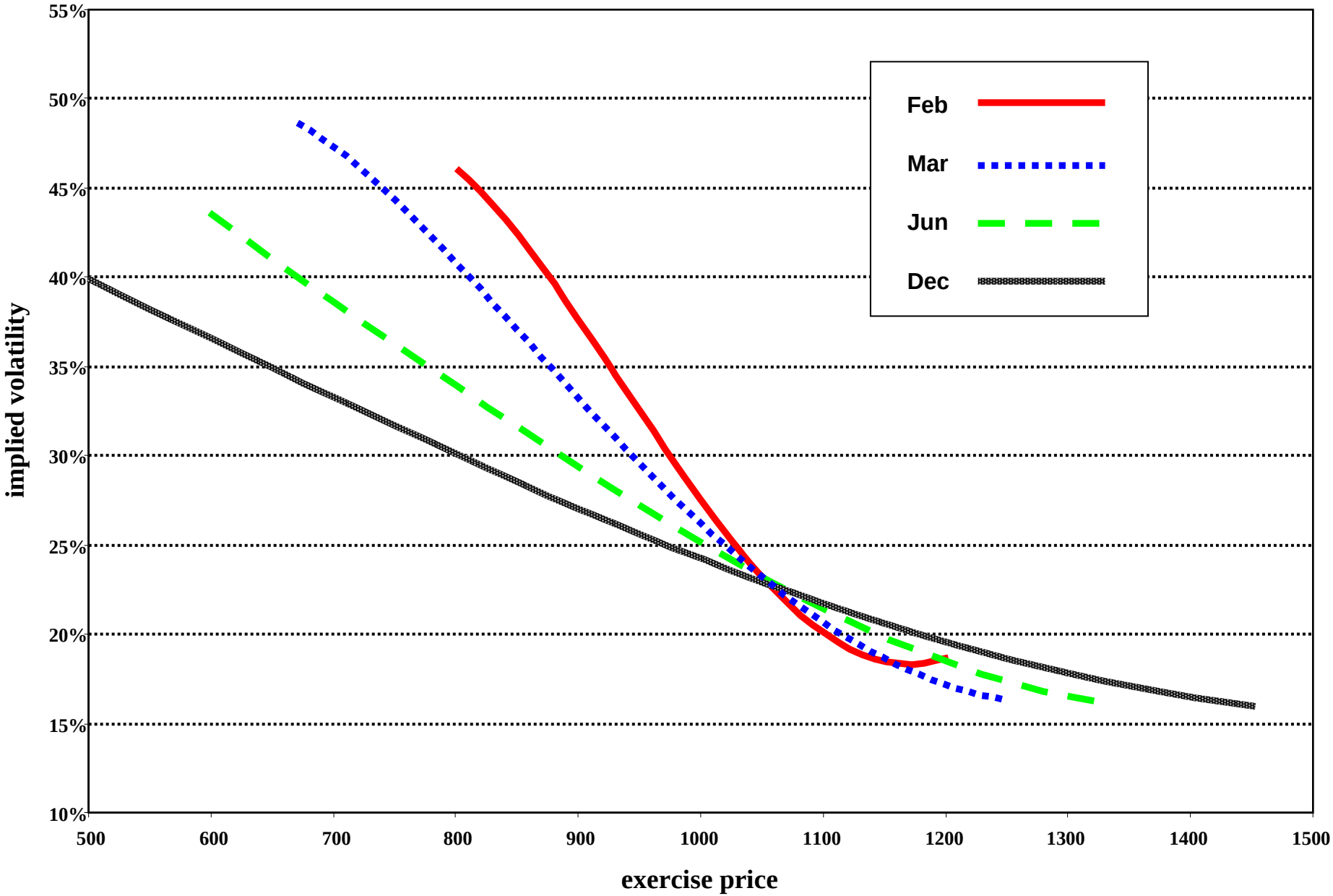
Is the 90 put further out-of-the-money with three months to expiration or one month to expiration?

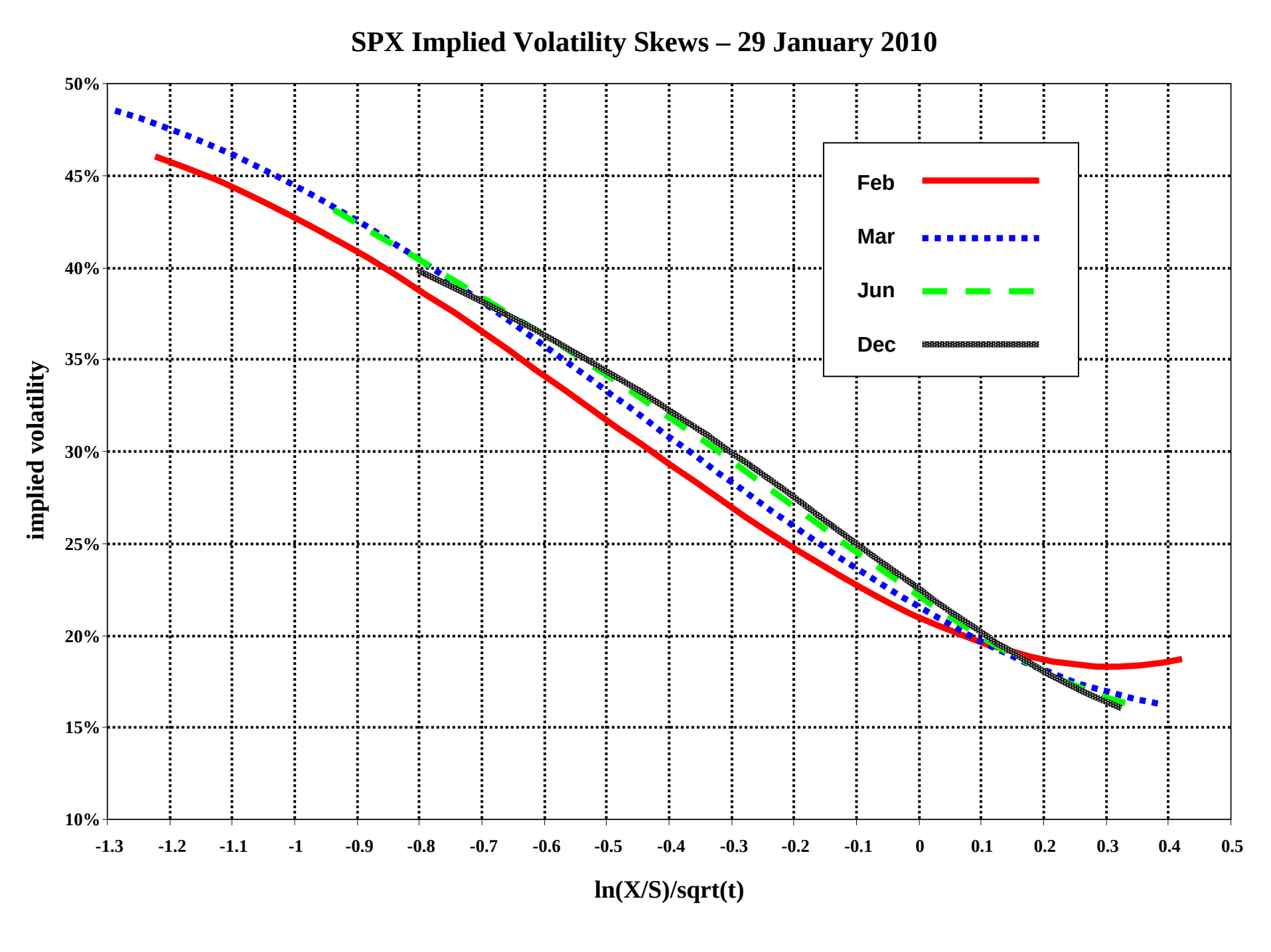


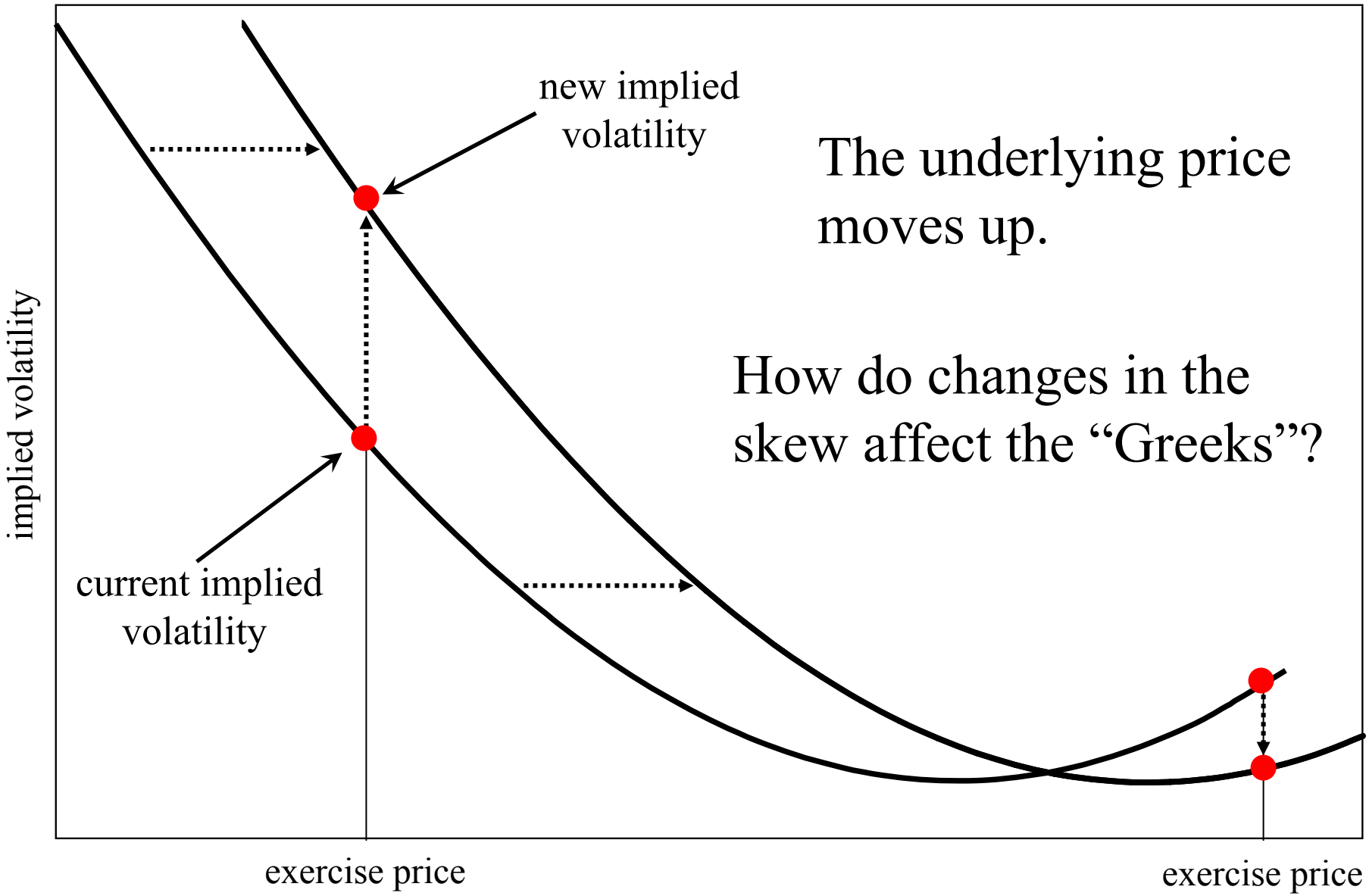

How will the passage of time affect the shape of the skew?
less time to expiration
more time to expiration
implied volatility
exercise price


How will the passage of time affect the shape of the skew?
less time to expiration
more time to expiration
implied volatility
ln (X / S) / sqrt (t)

SPX Implied Volatility Skews – 29 January 2010




SPX Implied Volatility Skews – 29 January 2010
50%
45%
40%
35%
30%
25%
20%
15%
10%
implied volatility
-1.3
-1.2
-1.1
-1
-0.9
-0.8
-0.7
-0.6
-0.5
-0.4
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
0.4
0.5
ln(X/S)/sqrt(t)
Feb
Mar
Jun
Dec


new implied volatility
The underlying price moves up.
How do changes in the skew affect the "Greeks"?
implied volatility
current implied volatility
exercise price
exercise price

underlying price = 100.00

95 put = 2.00

implied volatility = 26.0% implied delta = -25

underlying price rises to 101

95 put $\approx$ 2.00 - (.25 x 1.00) = 1.75

shifted implied volatility = 27.0%

vega of 95 put = .07

95 put $\approx$ 1.75 + (1 x .07) = 1.82

delta of 95 put = (1.82 - 2.00) / (101 - 100) = -.18

adjusted or ***skewed delta = -18***

underlying price = 100.00

105 call = 2.30

implied volatility = 21% implied delta = 17

underlying price rises to 101

105 call $\approx$ 2.30 + (.17 x 1.00) = 2.47

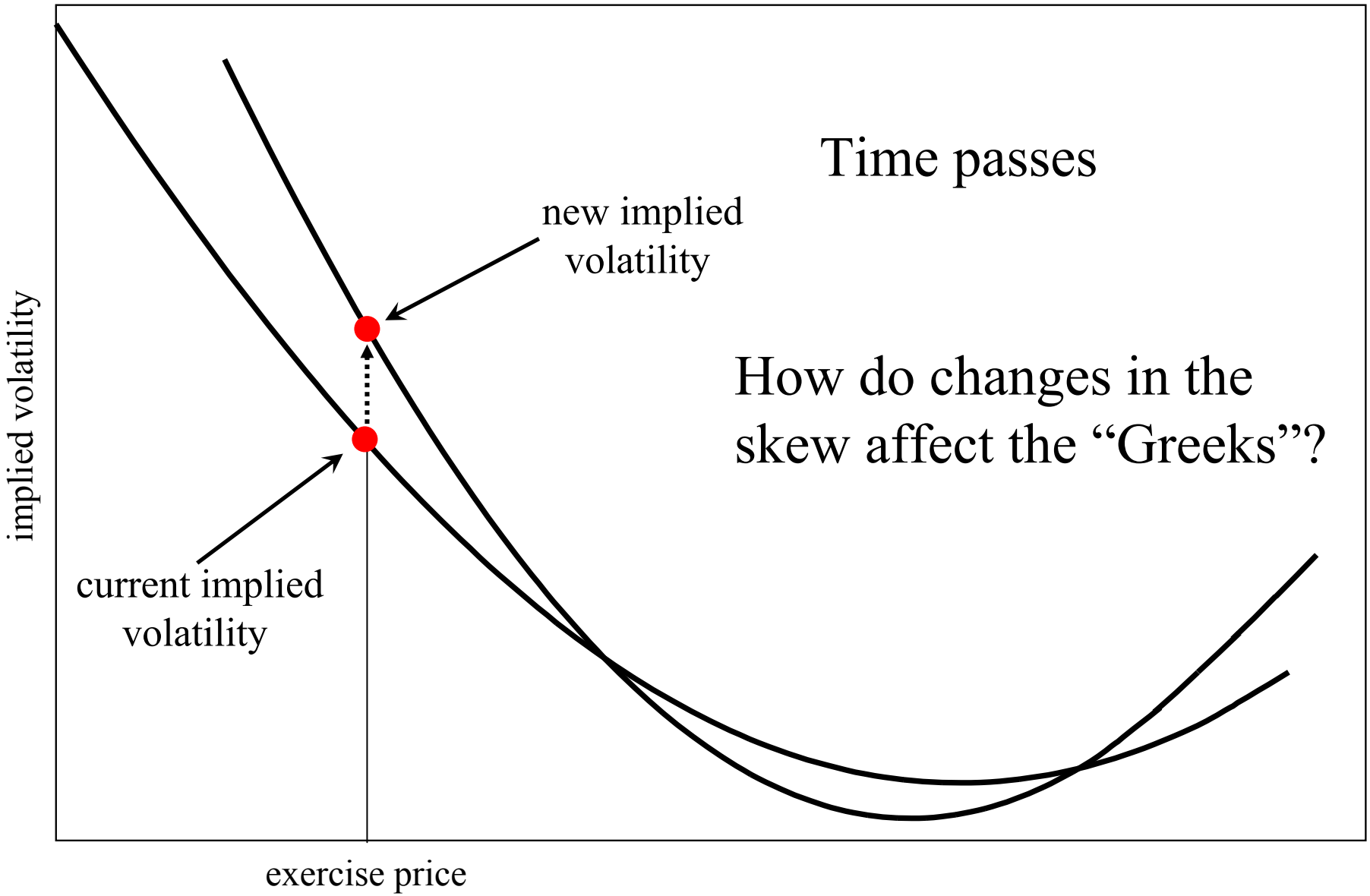
shifted implied volatility = 20.5%

vega of 105 call = .16

105 call $\approx$ 2.47 - (.5 x .16) = 2.39

delta of 105 call = (2.39 - 2.30) / (101 - 100) = .09

adjusted or ***skewed delta = 9***


Time passes
new implied volatility
How do changes in the skew affect the "Greeks"?
current implied volatility
implied volatility
exercise price

underlying price = 100.00

95 put = 2.00

implied volatility = 26.0% implied theta = -.08

One day passes

95 put $\approx$ 2.00 - .08 = 1.92

shifted implied volatility = 26.3%

vega of 95 put = .07

95 put = 1.92 + (.3 x .07) $\approx$ 1.94

theta of 95 put = (1.94 - 2.00) = -.06

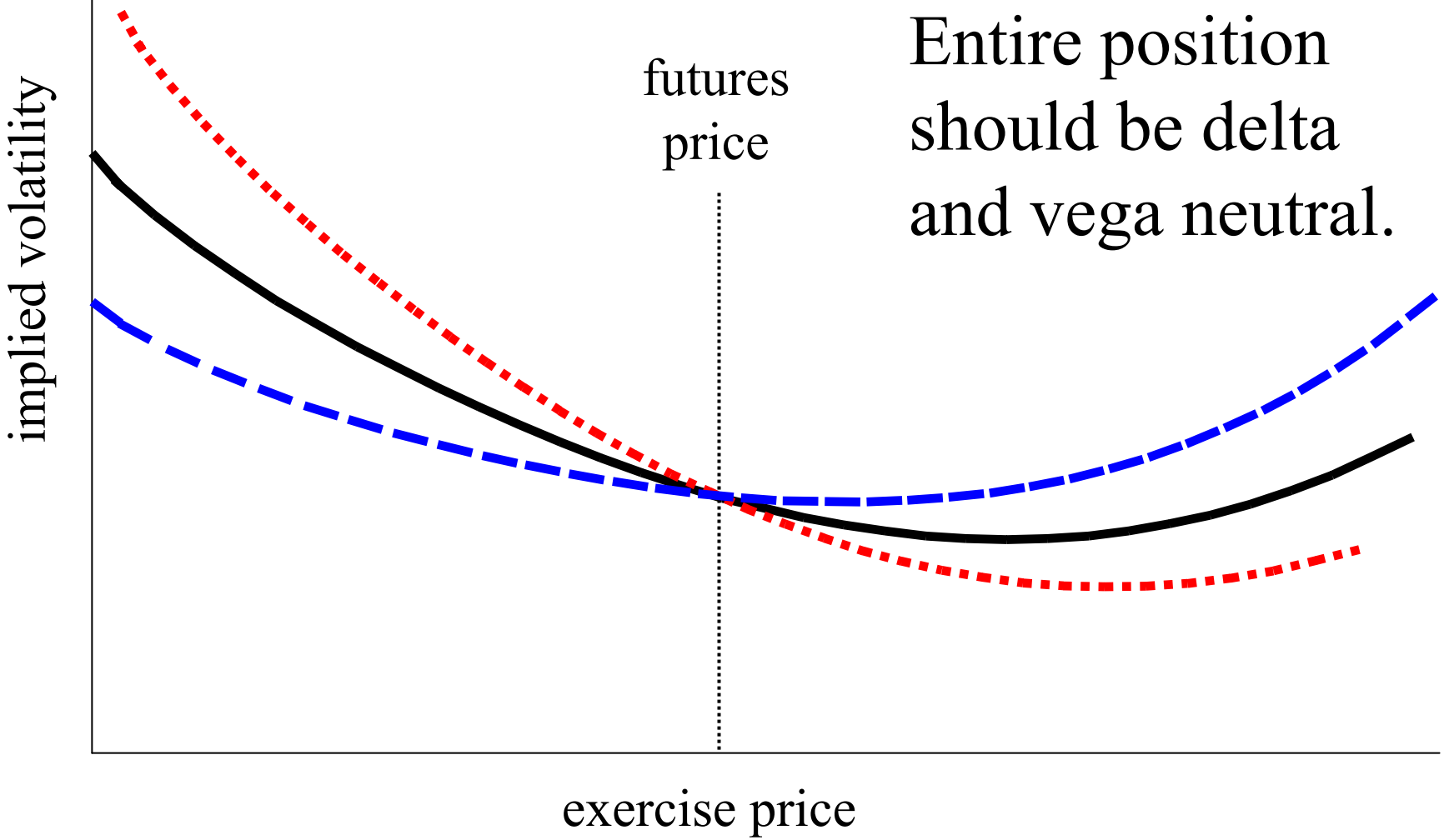
adjusted or ***skewed theta = -.06***

Theoretical Pricing Model

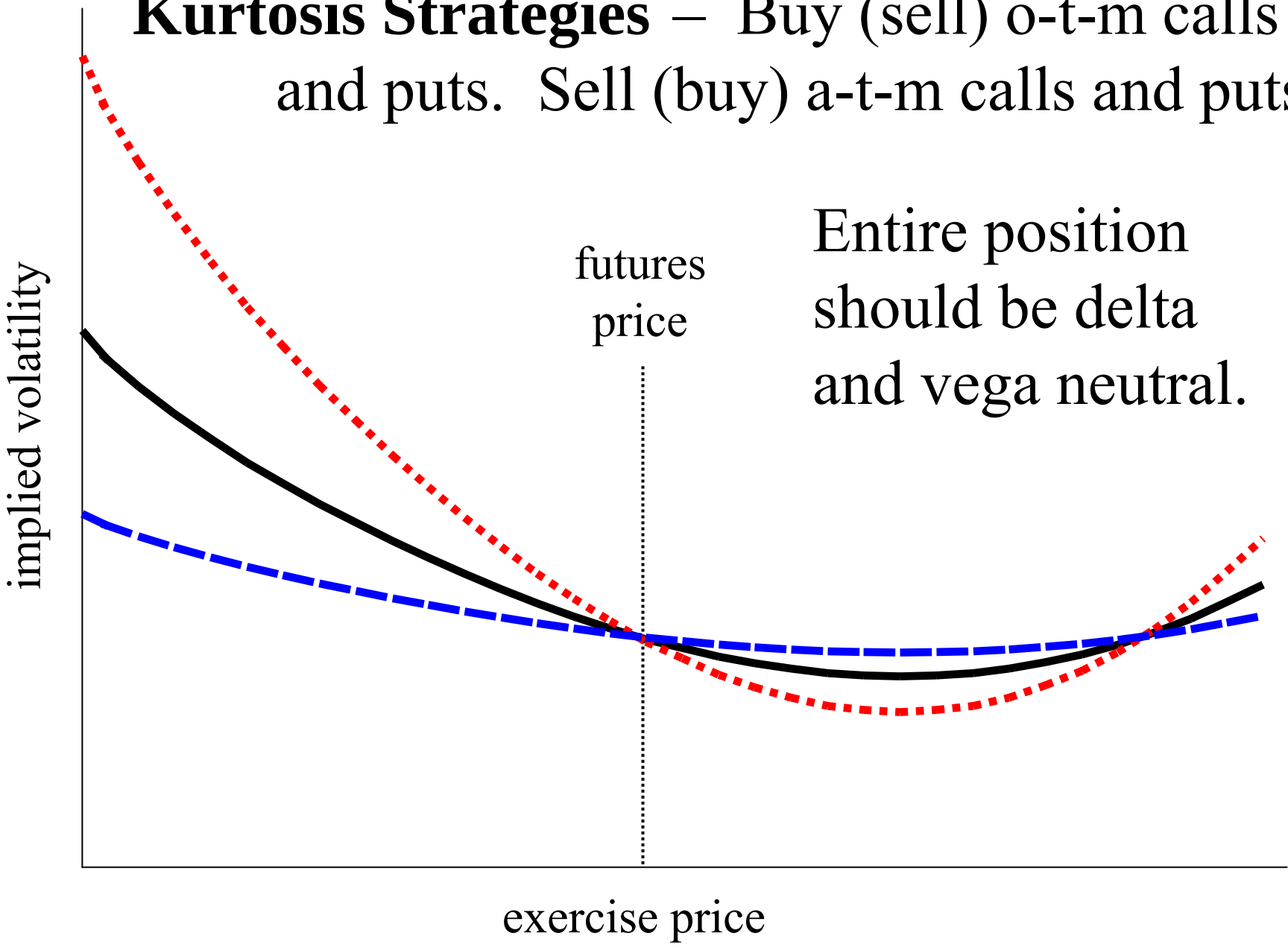
Exercise Price	
Time to Expiration	theta
Underlying Price	delta
Interest Rate	rho
Volatility	gamma / vega
Skew	***skew sensitivity***
	$f(x) \longrightarrow f(a,b,\ldots,x)$

Skew Strategies – Buy o-t-m calls (puts) and sell o-t-m puts (calls). Sell (buy) futures.

Entire position should be delta and vega neutral.



Kurtosis Strategies – Buy (sell) o-t-m calls and puts. Sell (buy) a-t-m calls and puts.



Entire position should be delta and vega neutral.